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ELECTROMAGNETIC SOIL PROPERTIES IN THE VHF/UHF RANGE (PHASE I)

by

Robert A. Falls
Andrew Cuneo, Jr.
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May 1972

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U. S. ARMY MOBILITY EQUIPMENT RESEARCH AND DEVELOPMENT CENTER
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13. ABSTRACT

This report describes measurements of the electromagnetic parameters of soil samples in the VHF/UHF range the purpose of which was to accumulate basic data on the properties that limit subsurface target detection by EM methods. A section of coaxial waveguide filled with soil is treated as a length of lossy transmission line. Classical transmission-line theory is used in determining the complex propagation constant of the soil sample from measurements on the electrical phase and the VSWR. These data are fed into classical equations which were programmed for computer solution.

Also included in the report for completeness and comparison are the results of two other techniques for measuring the real part of the complex propagation constant. The first of these techniques requires electrical phase and VSWR measurements and the solution is obtained graphically. The second technique requires the measurement of a power ratio under matched conditions of the line under test.

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May 1972

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The Commanding Officer
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Prepared by

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Mine Detection Division
Countermine/Counter Intrusion Department

SUMMARY

This report describes measurements of the electromagnetic parameters of soil samples in the VHF/UHF range made to accumulate basic data on the properties that limit subsurface target detection by electromagnetic (EM) methods. A section of coaxial waveguide filled with soil is treated as a length of lossy transmission line. Classical transmission-line theory is used in determining the complex propagation constant of the soil sample from measurements on the electrical phase and the voltage standing-wave ratio (VSWR). This data is fed into classical equations which were programmed for computer solution.

Also included in the report, for completeness and comparison, are the results of two other techniques for measuring the real part of the complex propagation constant. The first technique requires electrical phase and VSWR measurements, and the solution is obtained graphically. The second technique requires the measurement of a power ratio under matched conditions of the line under test.

FOREWORD

This work was done in support of Project 1J662712AJ22, Barrier Detection Research.

The following people have contributed to this report: Robert L. Brooke, who encouraged and suggested the "power ratio under matched condition technique," furnished the critical equipment, and served as a consultant; Benjamin Fletcher, who performed most of the early measurements using the "Ginzton Technique"; Walter J. Scott, who performed the "power ratio under matched condition technique" measurements; Ingrid Scharn, who programmed the equations for the "input impedance technique"; and Charles N. Johnson, Jr., for his review.

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IN THE VHF/UHF RANGE (PHASE I)

I. INTRODUCTION

1. **Purpose of Program.** The primary purpose of the initial phase of the program is to select measurement techniques and to measure and record the values of the electromagnetic properties of soils with a description of the methods used. The long-range objective of these measurements is to gain a clearer understanding of what role the soil environment plays in subsurface target detection.

2. **Known Measurement Problem.** The electromagnetic properties of a soil play an important role in determining whether subsurface targets can be detected. The preferred way for determining these properties would be to bring the laboratory to the sites and measure the soil in situ. This method is not the normal and practical way to measure soil properties, at least, not in the early stages of an investigative program. This method would limit the types of soils to the immediate laboratory location.

The second method, which is the most practical for the initial stages, is to obtain small samples of soil from many parts of the world and to bring them to the laboratory. It has the disadvantage that the soils are disturbed and properties thereby are possibly altered. However, this method offers the advantage of testing a large number of soil samples rather easily and the flexibility of a fully equipped laboratory in selecting the measuring techniques. The ultimate technique would incorporate the best laboratory method into a mobile, rapid procedure to test soil in situ.

II. INVESTIGATION

3. **Review of Known Measurement Techniques.** Various methods by which the electrical properties of soil can be measured can be grouped under two major headings: those that employ a radio ground wave and those that do not. The two groups are as follows:

A. Methods using radio ground waves (in situ measurements):

- Attenuation of Ground Wave
- Wave Tilt
- Magneto-Telluric
- Reflection Coefficient

B. Methods using signal generators, etc.:

- Electrode Array (in situ)
- Bridge Substitution
- Intrinsic (one-way) Loss of 4-Terminal Network:
  - Ginzton Technique
  - Power Ratio Technique
  - Kirthscether's Transmission-Line Method

The methods which use a radio ground wave for the measurement of the electrical properties of soil are all large scale, field in situ methods, whereas the second group contains methods which can be carried out in a much more limited area and, with the exception of the electrode array, can be carried out in the laboratory.

A summary of measurement techniques is given in Table I. A brief review of the known methods follows:

a. **Attenuation of Ground Wave.** The attenuation vs distance technique requires a high-power transmitter and extensive field strength measurements over a large area. Conductivity measurements of a specified small area are not possible with this method.

b. **Wave Tilt.** The wave tilt method is, perhaps, the most used method of determining the effective soil-conductive and dielectric-constant values. A transmitter, transmitting antenna, receiver, and two receiving antennas are included in the system. An electromagnetic field is produced by the transmitter. The amount of tilt of the electromagnetic wave across the surface is a function of the effective ground constants and frequency used.

c. **Magneto-Telluric.** This method measures the naturally occurring E/M field at the earth's surface to determine the surface impedance and subsurface strata. The subsurface has to be effectively homogeneous in a horizontal direction for distances much greater than the wavelength employed.

d. **Determination of Reflection Coefficient.** This method computes the conductivity and dielectric constant of a soil by propagating an electromagnetic wave between two nonconducting towers and by comparing the phase change between the direct and ground-reflected waves. Tower heights and separation have to be of the order of a wavelength or greater to avoid near-field phenomena.

Table I. Summary of Measurement Techniques

Location	Method	Properties Measured
IN SITU (Gross Measurements)	Attenuation of Ground Wave	Soil Conductivity/Dielectric Constant
	Wave Tilt	Effective Soil Conductivity/Dielectric Constant
	Magneto-Telluric	Apparent Surface Impedance
	Reflection Coefficient	Conductivity/Dielectric Constant
	Electrode Array	Conductivity
FIELD AND/OR LABORATORY METHODS	Bridge Substitution	
	Kirkscether's Transmission-Line	
LABORATORY METHODS	Intrinsic Loss: Ginzton	Attenuation
	Intrinsic Loss: Power Ratio	Attenuation
	Kirkscether's Transmission-Line	Conductivity, Dielectric Constant, Velocity of Propagation, Attenuation

e. **Electrode Array.** Wenner describes a four-in-line electrode method of measuring earth resistivity.[1] The measuring technique is simple. The current source is applied to two electrodes, and a voltage is read on another pair of electrodes. The soil conductivity can be computed from a formula.

f. **Bridge Substitution-Resistivity.** The resistivity (low-audio-frequency) bridge method is generally employed in the laboratory to obtain the electrical properties of soil samples. It can, however, be used in the field to obtain in situ measurements. The relative dielectric constant and conductivity can be obtained with this method. Some disadvantages are: the surface reactance between the electrodes and soil; and the breaks, cracks, or discontinuities produced in the soil by the insertion of the electrodes. These can lead to errors in the value of the properties.

g. **Intrinsic (One-way) Loss of a 4-Terminal Network.**

(1) **Ginzton Technique.** The intrinsic loss of a soil packed in a coaxial line can be found by measuring the input impedance of the network for a series of positions of a movable short circuit at the output terminals. This is a laboratory method and reveals only the attenuation at frequencies at and above 100 mHz. Details are given in paragraph 4a.

(2) **Power Ratio Technique.** This technique is based on the fact that the one-way loss through a network containing a soil sample matched both at the input and output is given by a simple equation. This method is faster and less prone to human mistakes than the Ginzton Technique. It is a laboratory method only. Details are given in paragraph 4b.

(3) **Transmission-Line Method (Kirkscether).** The soil sample is packed in a suitable transmission line (coaxial line). An adequate length of the soil is selected (15 to 30 centimeters). The input impedance of the line with the soil is measured with the far end of the sample line short circuited, then open circuited. The method can be used to determine the conductivity, dielectric constant, attenuation, and velocity of propagation. This method has very good possibilities for field in situ measurements. Details are given in paragraph 4c.

4. **Methods Utilized in This Investigation.**

a. **Ginzton Technique.** The first in-house attempt to measure attenuation (of soil) employed a technique described by Ginzton.[2] According to Ginzton, "The

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[1] F. Wenner, "A Method of Measuring Earth Resistivity," Bulletin of the Bureau of Standards, Vol. 12 (1915).

[2] E. L. Ginzton, *Microwave Measurements*, pp. 465, 473, 474, McGraw Hill Book Co., Inc., New York, 1957.

intrinsic (one-way) loss, L, of an arbitrary four-terminal network (coaxial line) can be found by measuring the input impedance of the network for a series of positions of a movable short circuit at the output terminals..." It should be emphasized that the determination of the intrinsic loss, L_i, by this method does not require the network to be matched at either end. The necessary data can be taken for a series of positions of the movable short circuit. The position of the short circuit need not be measured. If input impedance locus is plotted on a Smith Chart, the intrinsic loss of the network, L_i, is given by:

$$L_i = 10 \text{ Log} \frac{\sqrt{(1+R)^2 - P^2} + \sqrt{(1-R)^2 - P^2}}{\sqrt{(1+R)^2 - P} - \sqrt{(1-R)^2 - P^2}} \text{ (db)}$$

where R is the radius of the impedance circle and p is the distance from the center of the impedance circle to the center of the Smith Chart. R and P are normalized to unity with respect to the center of the Smith Chart. In our case, the 4-terminal network is the soil-filled section of coaxial line, and (X = L/sample length). The actual coaxial receptacle for the solid was fabricated from 3/4-inch ID by 1/16-inch wall thickness brass pipe with a 0.322-inch-diameter brass center conductor. Alford #11890 reducers were used on both ends of the pipe. The rest of the system was made up of a signal generator, a low-pass filter, a pad, a slotted line, a VSWR meter, an adjustable line, and a short circuit.

b. **Power Ratio Under Matched Condition Technique.** Another technique for measuring soil attenuation was the power ratio under matched condition technique. A block diagram of the setup is shown in Fig. 1. The theory behind the technique is based on the fact that the one-way loss through a network which is matched both on its input and output ports is given by the simple equation,

$$L_i = 10 \log \frac{P_{out}}{P_{in}} \text{ (db)},$$

where P_{in} is the input power to the network and P_{out} is the power delivered to a matched load, and again L_i = L/sample length. These two power levels are measured by using directional couplers on the input and output of the network (coaxial line filled with soil). The network is matched on its input and output by using stub tuners and adjusting them for maximum power transfer. The detectors are matched over a frequency range which includes the range of interest. An Alford Network Analyzer Model 7051 is used to take the ratio of the detected levels.

The Power Ratio Technique using the Vector Voltmeter HP 8405A is a variation of the previously mentioned method. Essentially, the Alford Analyzer and

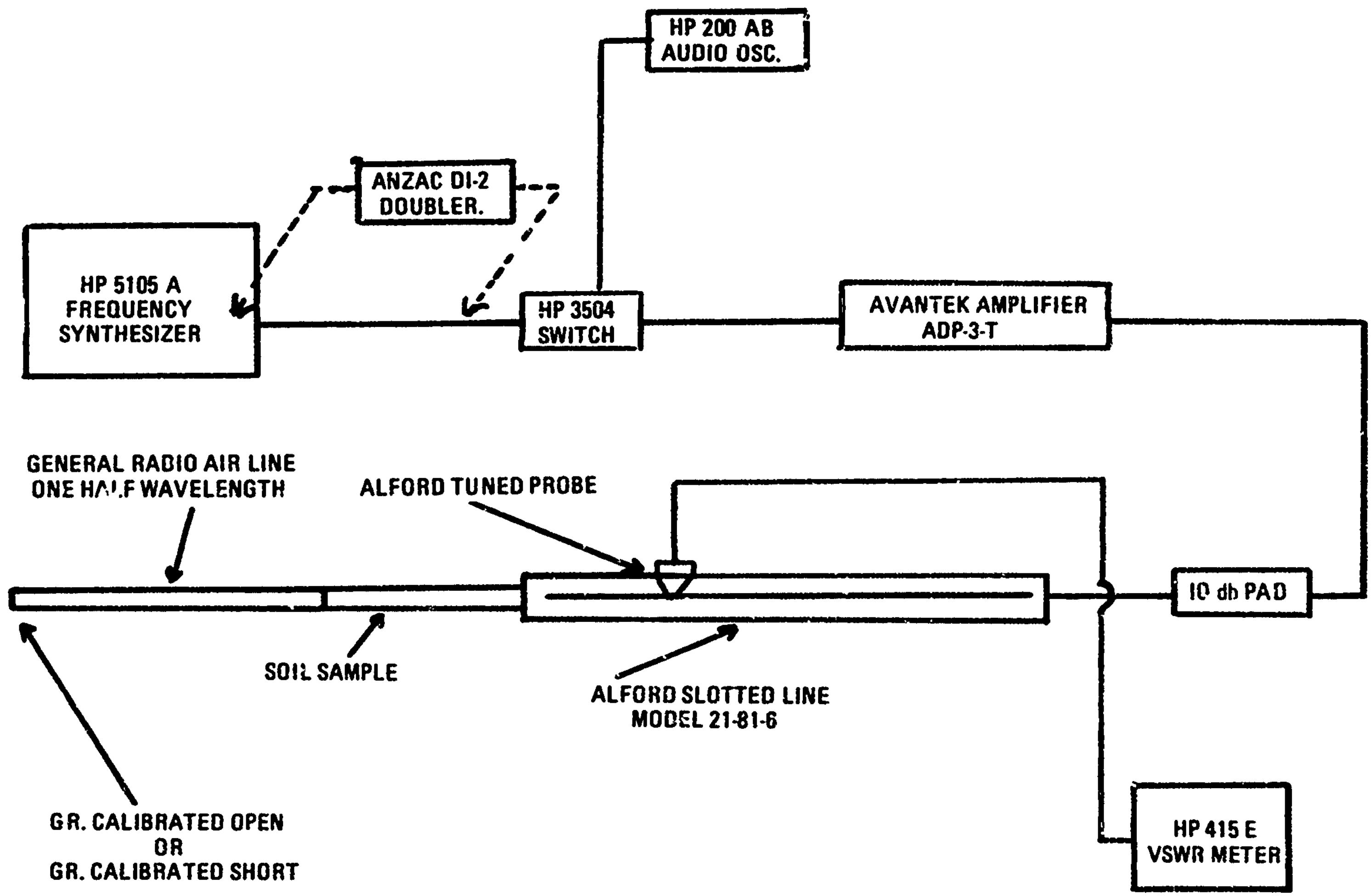

Fig. 1. Laboratory setup for measuring EM parameters of soil.

detectors are replaced by the Hewlett Packard Vector Voltmeter. The advantage of using the Vector Voltmeter is that it provides not only the direct readout of attenuation on a meter but also yields the phase angle of the sample. This method for soils measurements was originally conceived in the latter part of 1970.

The attenuation values obtained from this method correlate closely with those values obtained by an independent contractor laboratory and by the Input Impedance Technique.

c. **Kirkscether's Transmission-Line Technique.** The first attempt to measure the electromagnetic properties of dielectrics by inserting the material into a waveguide structure was reported by S. Roberts and A. von Hipple in 1946.[3] The measurements were made at the centimeter wavelengths in a rectangular waveguide. It was pointed out in the paper by Roberts and von Hipple that "by limiting the electromagnetic field to the closure of a hollow pipe or coaxial line, all boundary and stray effects disappear automatically and small amounts of any dielectric can be measured with precision." The values of the properties obtained using such a technique must be the actual free-space values in order for the technique to be useful. In a paper by T. W. Dakin and C. N. Works, use is made of the standing-wave measurement technique developed by Roberts and von Hipple and it is stated by Dakin and Works that, "In actual practice, the wave and the dielectric sample are restricted to an enclosed hollow or coaxial waveguide, although this is not in principle a necessary restriction. The same equations are valid in principle for a measurement using lecher wires or free space with a parallel beam of radiation, although it is more difficult in practice to do measurements under those conditions."[4] In the current report, classical theory, as stated in a paper by E. J. Kirkscether,[5] is employed.

According to theory, the propagation constant of a length of transmission line can be determined by a knowledge of the input impedance of the line with the output open circuited (Z_{oc}), then short circuited (Z_{sc}). The input impedance of the transmission line is related to the VSWR and the position of the voltage minimum of the standing-wave pattern setup on the input side of the transmission-line section under consideration. The attenuation (α) of a line is solved using the equation:

$$\alpha = \frac{1}{4\ell} \ln \left[(Z_{sc}, Z_{oc})\, f \right].$$

---

[3] S. Roberts, A von Hipple, "A New Method for Measuring Dielectric Constant and Loss in the Range of Centimeter Waves," *J. App. Phys.*, *17*, 610 (1946).

[4] T. W. Dakin, C. N. Works, "Microwave Dielectric Measurements," *J. App. Phys.*, *18*, 789 (1947).

[5] E. J. Kirkscether, "Ground Constant Measurements Using a Section of Balanced Two-Wire Transmission Line," *IRE Trans on Ant. and Prop.*, *AP-8*, 307 (1960).

The phase constant (β) is solved by using the equation

$$\beta_n = \frac{1}{2\ell} \left\{ \tan^{-1} [g (Z_{sc}, Z_{oc})] + n\pi \right\}.$$

By use of the instrumentation shown in Figs. 1 and 2, measurements can be made on a coaxial transmission line which is partially filled with a sample of soil whose EM properties are to be found.

The soil is prepared by packing it in a coaxial line (20 or 30 cm long) with the device shown in Figs. 3, 4, 5, and 6. The connectors are replaced, and the line containing the soil is connected to the slotted line as shown in Fig. 1. An adjustable, coaxial air line of exactly one-half wavelength is placed behind the sample. A calibrated short is placed on the end of the one-half-wavelength line and the VSWR reading, and the distance from the first null to the sample input is measured and recorded. The calibrated short is replaced with a calibrated open, and the procedure is repeated.

The four measurements and the frequency at which they were taken are programmed into a computer to obtain the following properties: attenuation, phase constant, velocity of propagation, dielectric constant, and conductivity (see Appendix A).

A detailed procedure for this technique can be found in Appendix B. The mathematical solutions to the transmission-line equations can be found in Appendices C and D.

5. Sources of Error (Kirkscether's Technique).

a. Presence of TE, TM Modes. The presence of higher modes (TE, TM) will invalidate all equations used, since they are derived from Maxwell's equations on the assumption of TEM. It will be shown b ow that, for the frequency range used, TE and TM modes cannot exist either in the air-filled or soil-filled coaxial line.

First, we shall show that even for the case of very lossy soil the wavelength is given very accurately by the simple relationship (for good dielectrics),

$$\lambda_s = \frac{\lambda_o}{\sqrt{\epsilon_r}}$$

where λ_s is the wavelength in the soil, λ_o is the wavelength in air, and ϵ_r is the relative

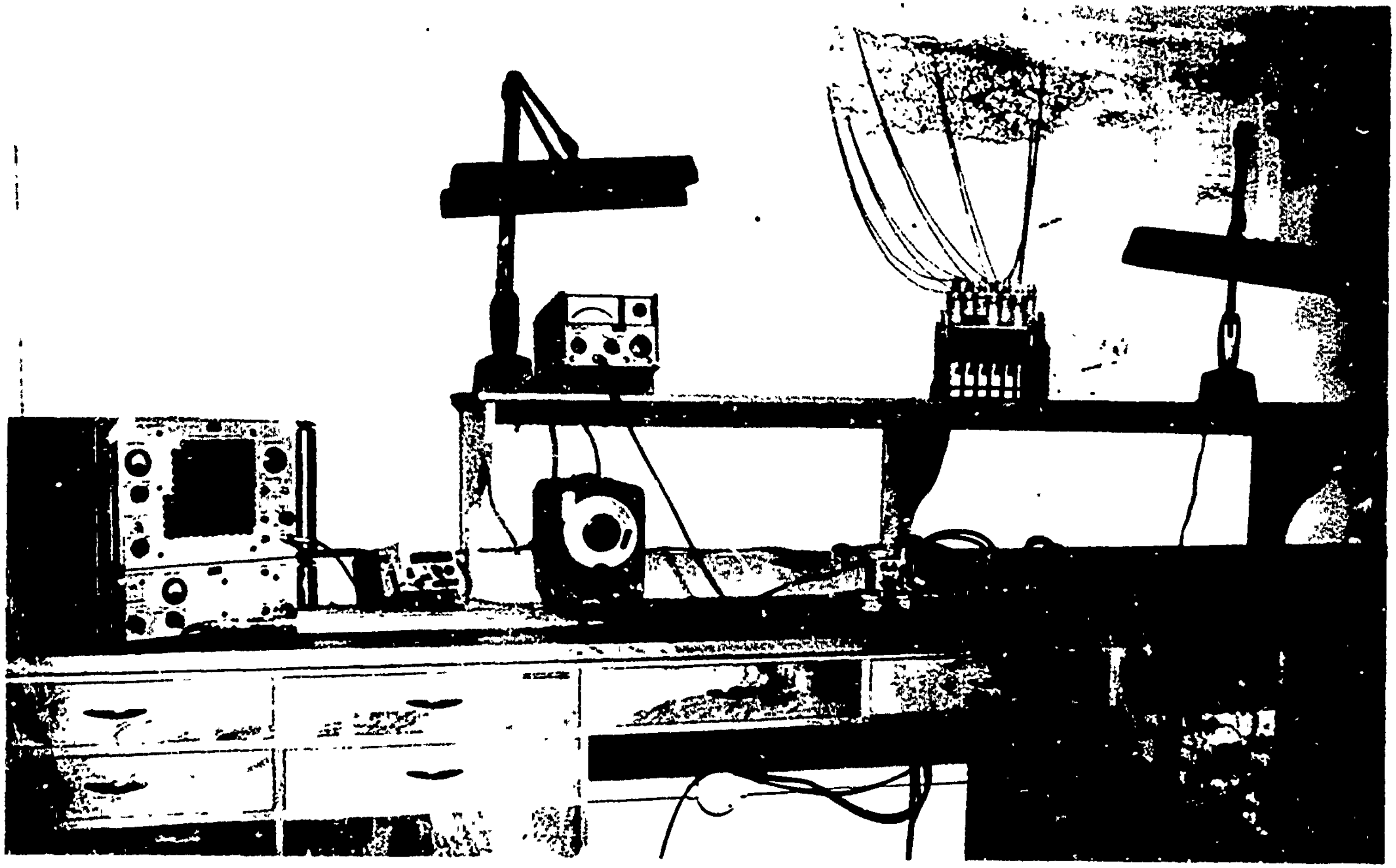

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Fig. 2. Laboratory equipment.

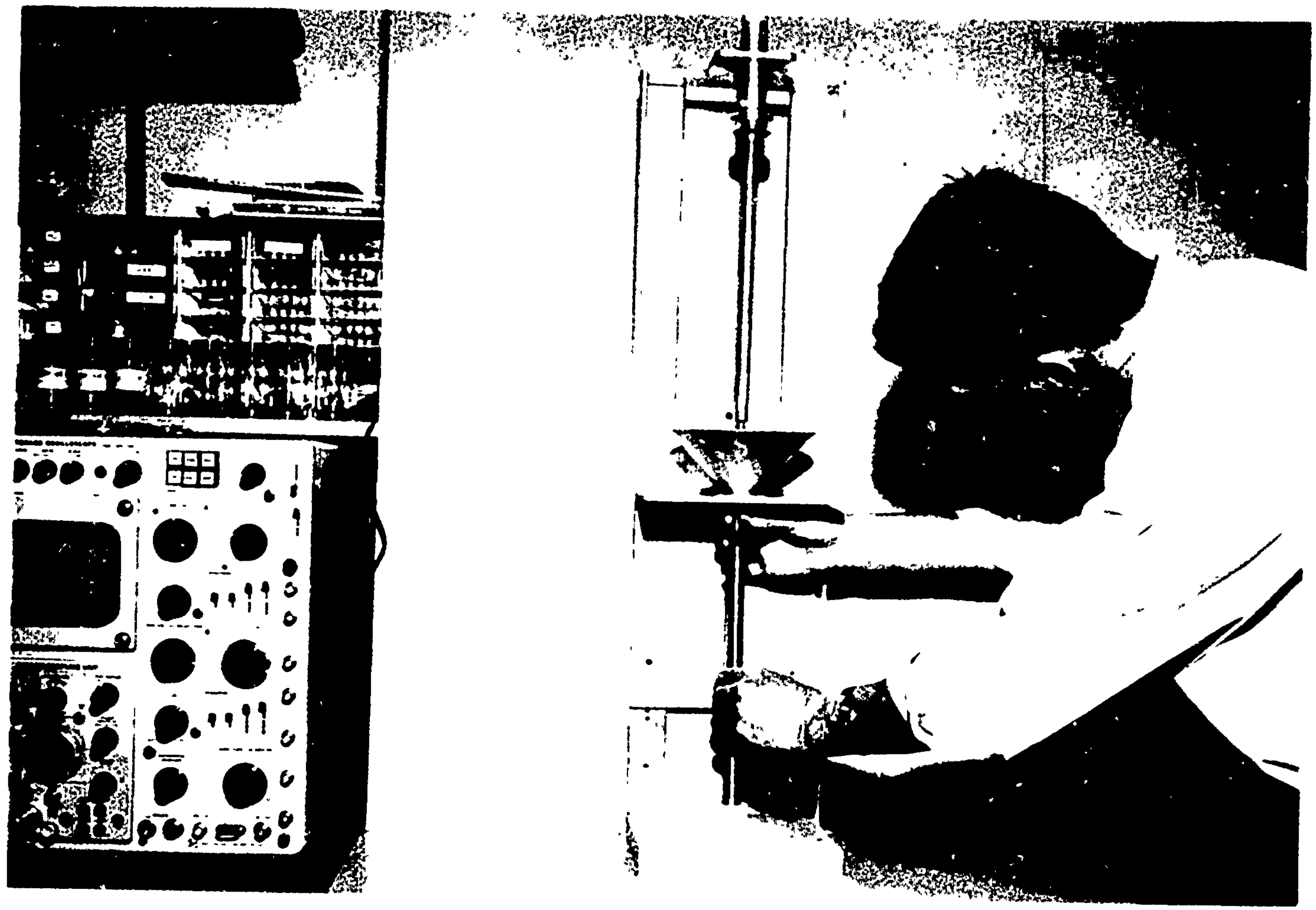

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Fig. 3. Coaxial line soil-insertion device.

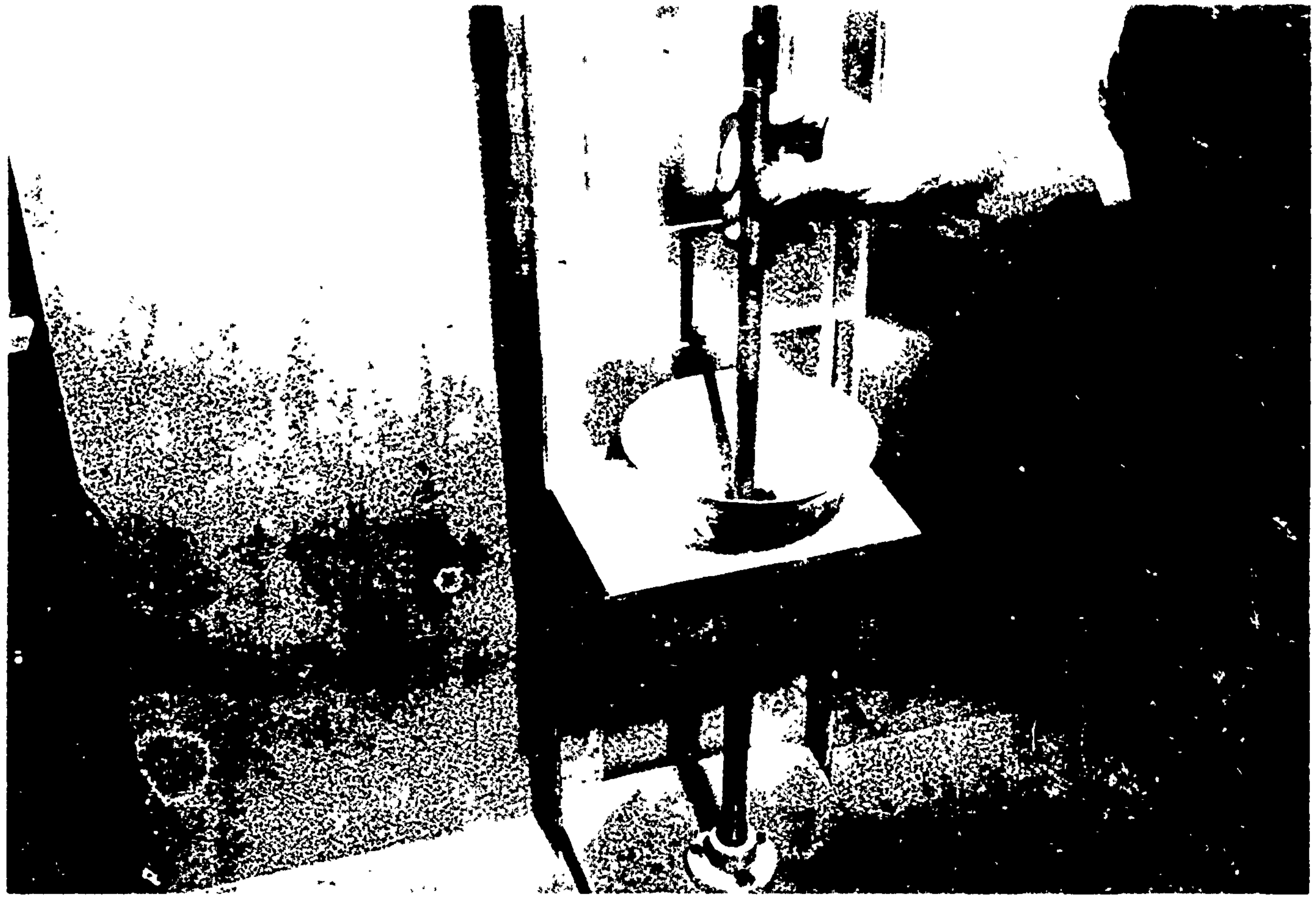

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Fig. 1 Setting up insertion device — tamping soil into air line.

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Fig. 5. Tamping soil into air line.

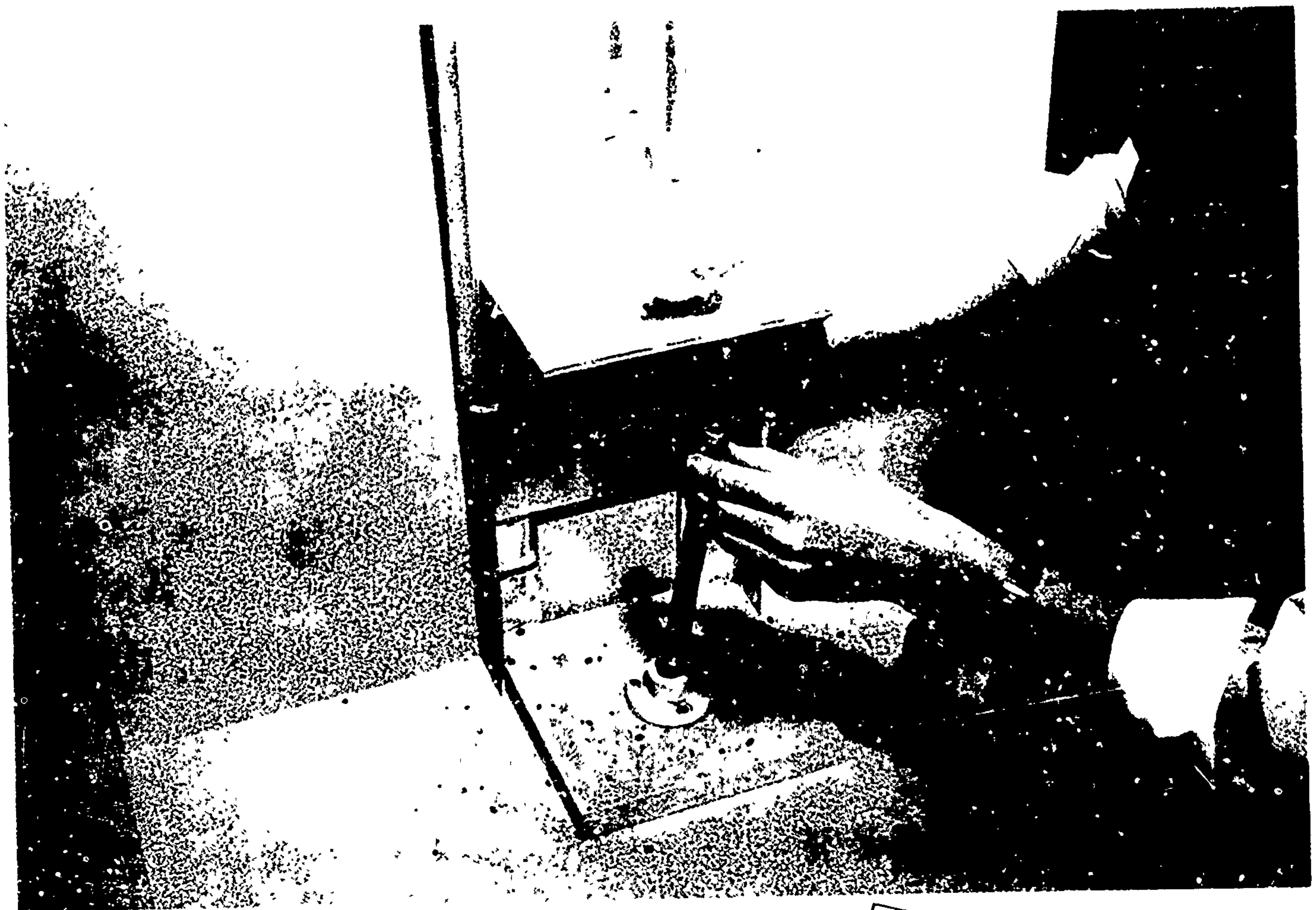

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Fig. 6. Removing tilted coaxial line

dielectric constant of the soil, Von Hipple, in 1954, reported for a clay soil (20% moisture content) at 300 mHz a value of loss tangent, δ, equal to 0.5 and dielectric constant, ϵ_r, equal to 20.0. Using these values, we can compute beta (and, therefore, the wavelength) in the soil since this case represents an extremely lossy soil. Starting with the basic relationship for beta,[6]

$$\beta = \omega \left[\frac{\mu \epsilon}{2} \left(\left\{ 1 + \tan^2 \delta \right\}^{1/2} + 1 \right) \right]^{1/2}$$

and inserting the value for the loss tangent we obtain,

$$\beta = \omega \left[\frac{\mu \epsilon}{2} \left(\left\{ 1 + (0.5)^2 \right\}^{1/2} + 1 \right) \right]^{1/2}$$

$$\beta = \omega \left[\frac{\mu \epsilon}{2} \left(\left\{ 1.25 \right\}^{1/2} + 1 \right) \right]^{1/2}$$

$$\beta = \omega \left[\frac{\mu \epsilon}{2} (2.12) \right]^{1/2}$$

$$\beta = 1.03\, \omega \sqrt{\mu \epsilon} \sqrt{\epsilon_r}$$

$$\beta = \frac{1.03}{c} \sqrt{\epsilon_r}$$

which can be written approximately as,

$$\beta \approx \frac{\omega}{c} \sqrt{\epsilon_r}$$

$$\beta \approx \frac{2\pi}{\lambda_o} \sqrt{\epsilon_r}$$

$$\beta \approx \frac{2\pi}{\lambda_o / \sqrt{\epsilon_r}}$$

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[6]Simon Ramo, John Whinnery, *Fields and Waves in Modern Radio*, p. 306, John Wiley and Sons, Inc., New York, 1960.

From the above expression, we see that the wavelength in the soil is given by $\lambda_o/\sqrt{\epsilon_r}$ since beta is by definition equal to 2π/wavelength.

In order to insure that no higher mode will be propagated, the wavelength in the medium in question must obey the inequality.[7]

$$\lambda > \pi (b + a)$$

where b is the radius of the outer conductor and a is the radius of the inner conductor. For the coaxial line being used, b = .007 meter and a = .003 meter. Substituting these values in the above equation,

$$\lambda > \pi (0.007 + 0.003)$$

$$> 3.14 (0.01)$$

$$> 0.03 \text{ meter}$$

$$\lambda = \lambda_s = \frac{\lambda_o}{\sqrt{\epsilon_r}}$$

$$\lambda_o = 0.03 \sqrt{\epsilon_r} \cdot \text{meter}$$

and setting $\epsilon_r = 20$ (probable upper limit for typical soils),

$$\lambda_o = 0.03 \sqrt{20} = 0.03 (4.47)$$

$$= 13.4 \times 10^{-2}$$

$$\lambda_o = 0.134 \text{ meter.}$$

This corresponds to a frequency of 2.24 gHz. Under the assumption of $\epsilon_r = 20$, the measurements would be valid up to a frequency of 2.2 gHz. Our measurements went no higher than 1.0 gHz.

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[7]Theodore Moreno, *Microwave Transmission Design Data*, p. 69, Dover Publications Inc., New York, 1958.

b. Inherent Errors Due to Equipment. When calibrated terminations connected to the output end of the Slotted Line were used, the accuracy of the instrument was checked over a VSWR ranging from 1.00:1 to 6.00:1. The maximum error found was 5%. The actual components (Fig. 7) connected to the end of the Slotted Line have a maximum collective VSWR of (1.01) (1.01) (1.03) = 1.05.[8] The estimated effect that this VSWR has on measurements is given in Table II.

Table II. Errors Due to Equipment

VSWR of Sample in Ideal Line (No Inherent VSWR)	VSWR Range of Sample (Due to Inherent VSWR of Line)	Error
1.5	1.44 – 1.56	± 4%
2.0	1.92 – 2.08	± 4%
4.0	3.84 – 4.16	± 4%
6.0	5.76 - 6.25	± 4%

When the 5% error of the slotted line itself is considered, the total estimated maximum error turns out to be ± 9%. Since calibrated mismatches were available only up to 6.00:1, no calibration of the slotted line was made above this value. However, standard, accepted procedures for measuring high VSWR's (greater than 10.00:1) were followed. The above technique (Kirkseether's) yields the value of a number of electromagnetic parameters, i.e., dielectric, propagation and phase constants as well as attenuation and permeability. The techniques described in paragraph 6 yield only the value of attenuation.

6. Discussion of Techniques and Results. The advantages and disadvantages of the three techniques and the results of tests described in this report are given in the following paragraphs.

a. The Ginzton Technique. The Ginzton Technique was the first attempt at MERDC to measure the one-way attenuation through soil. This technique required approximately nine VSWR readings on one sample at one frequency. The readings were plotted on a Smith Chart, and impedance values from the chart were tediously hand calculated to arrive at the attenuation.

The Ginzton Technique is a laborious method to obtain the attenuation value of a sample. The hand calculations can easily lead to mistakes in the final answer. Soil attenuation vs frequency for this method appears in Fig. 8.

[8] *Microwave Engineer's Technical and Buyers Guide*, p. 11, Horizon House, Mass., 1967.

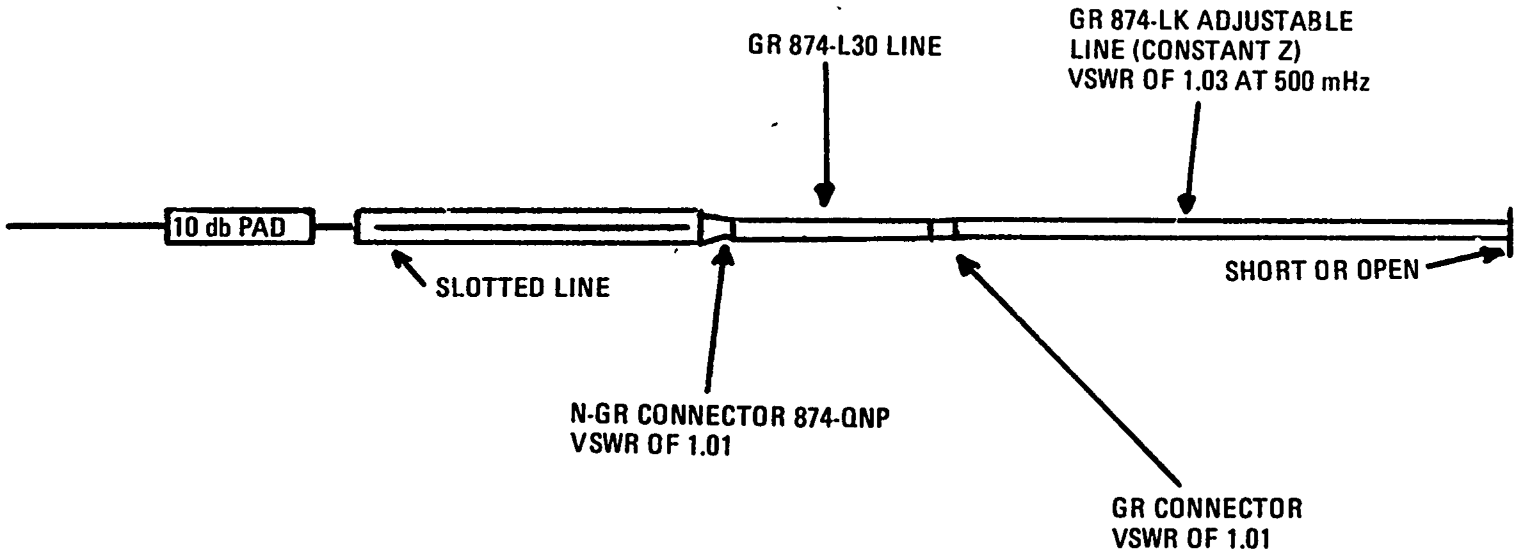


Fig. 7. Inherent system mismatches.

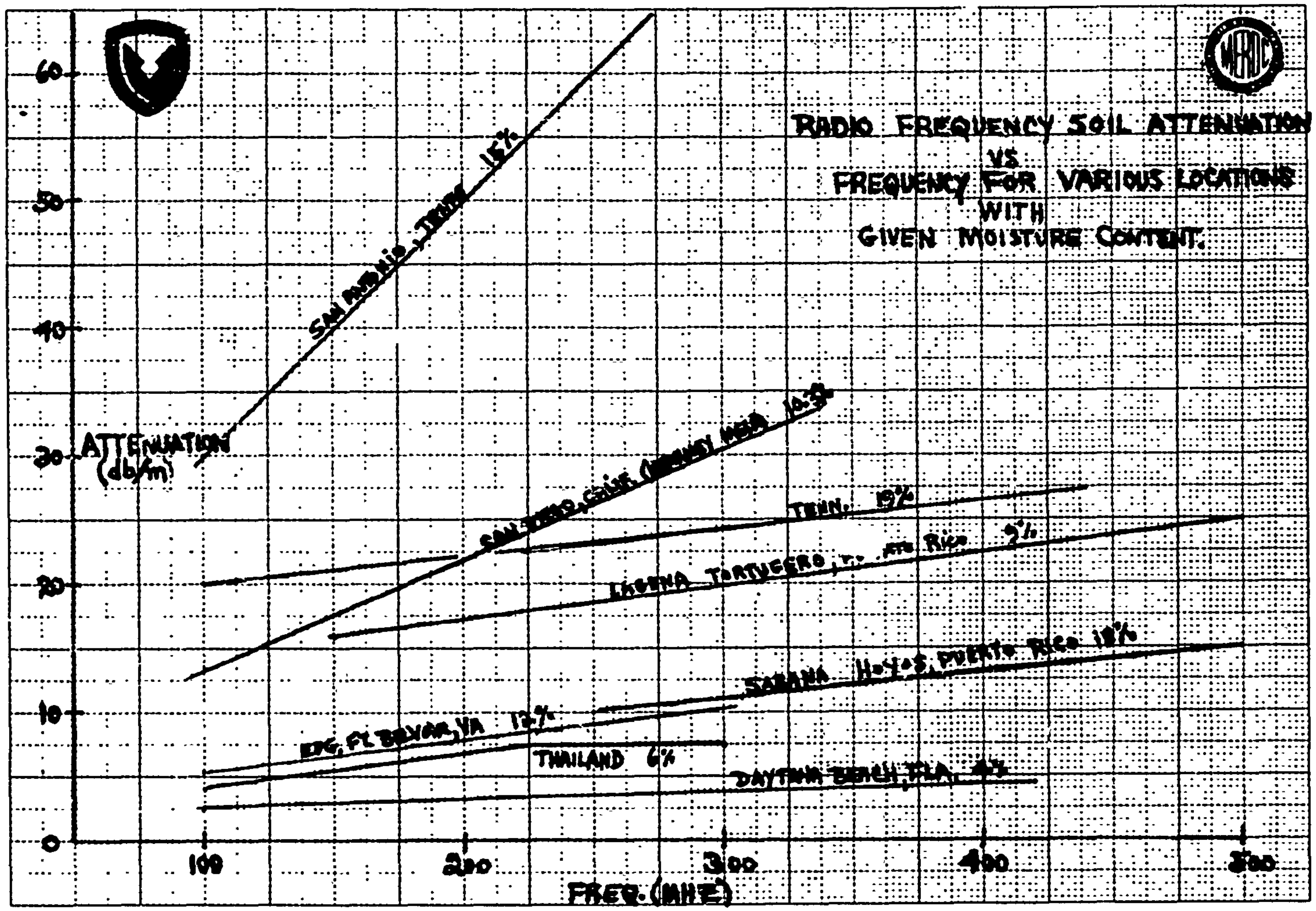


Fig. 8. Attenuation vs frequency (Ginzton Technique).

b. **The Power Ratio Technique.** The Power Ratio Technique using the Alford Network Analyzer was the second method employed in the quest for attenuation data from soils. This technique requires that the soil sample input and output ports be tuned for maximum power transfer as indicated on a network analyzer oscilloscope (Fig. 9). Once this is done, the measurement of the attenuation is obtained by directly reading, in decibels, the control knob on the analyzer.

This method is the least laborious and has the minimum calculations, necessary to arrive at an attenuation value. It has the disadvantage of not being able to go below approximately 190 mHz because of the short length of the tuners which are used to match the impedance of the line to the soil sample.

c. **The Input Impedance Technique.** The Input Impedance Technique was employed as a method to obtain not only attenuation but also velocity of propagation, conductivity, and dielectric constant. This technique, therefore, had greater potential in yielding more properties and their values from one initial set of readings. However, except for the attenuation, the other properties proved to be more elusive than at first thought. The equation finally used to obtain these properties proved to be multi-valued: and, unless the experimenters had previous knowledge of the approximate dielectric value of the soil under test, there was no way to identify the properties except by using two different lengths of the same soil.

d. **Results.** In weighing the results and potential (yielding other properties besides attenuation) of each technique against the effort, a conclusion can be reached as to which is the most effective technique and which is the least effective.

Assuming that the shortcomings of the Input Impedance Technique are solved (Phase Constant, ρ) Table III ranks the techniques in effort vs results.

Table III. Effort vs Results

Technique	Rank	Effort	Results	Use for Other Soil Measurements	Remarks
Ginzton	Fair	Tedious	Attenuation	None	--
Power Ratio	Good	Low	Attenuation	None	--
Power Ratio with Vector Volt	Very Good	Average	Attenuation w/Phase A	Dielectric Constant	
Input Impedance	Very Good to Best	Tedious	Attenuation	Phase β, Vel. of Prop., Dielectric Constant	Not good for high loss soils

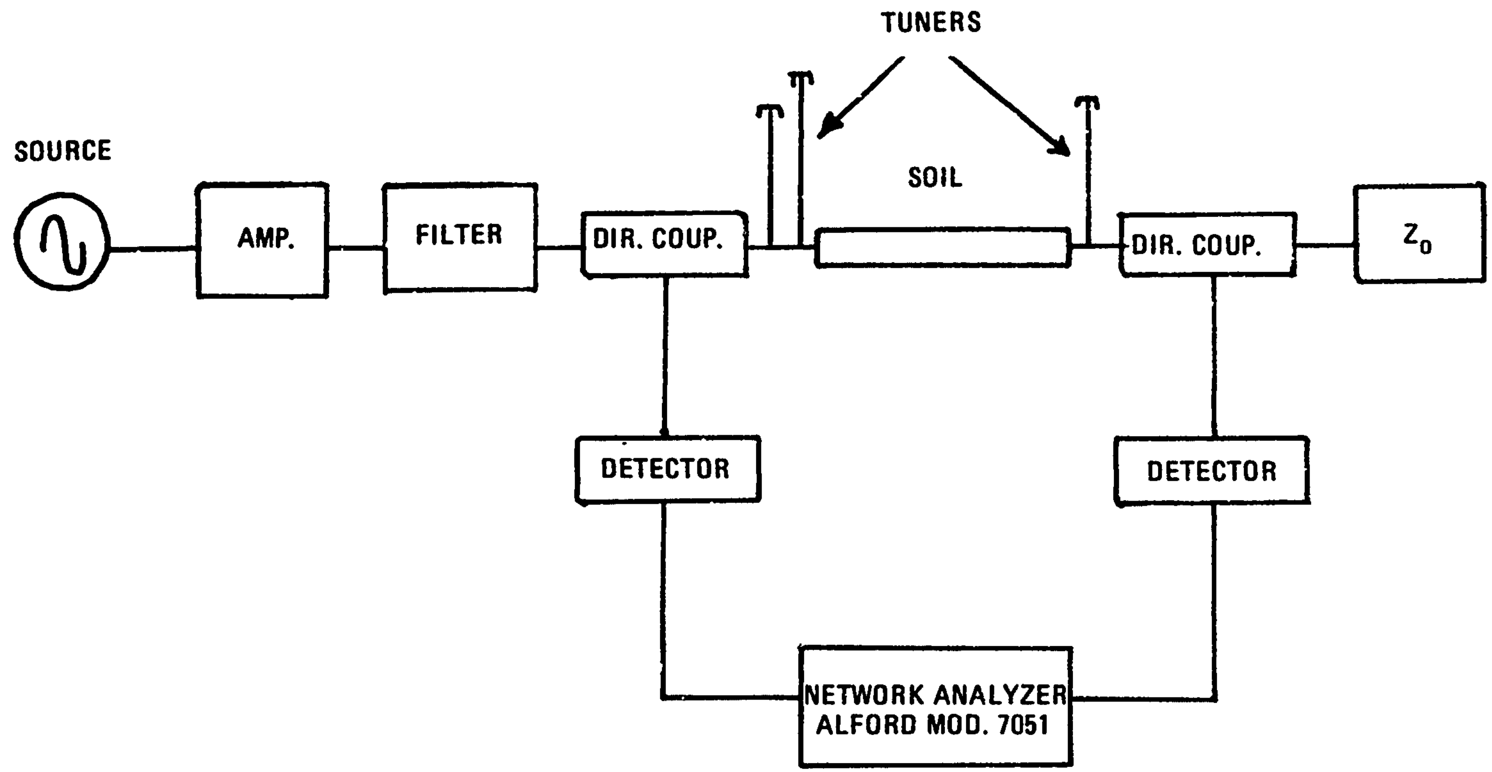


Fig. 9. Power ratio measurement under matched conditions technqiue.

There is good correlation between the Power Ratio and the Input Impedance Techniques as indicated by Figs. 10 and 11. Table IV shows the results of a recent comparison between the Power Ratio (Vector Voltmeter) Method versus the Input Impedance Technique on the same sample of soil at two different frequencies (i.e., density and moisture content were identical in the samples for both techniques).

Table IV. Comparison of Power Ratio/Input Impedance Techniques

Frequency (mHz)	Moisture % of Dry Wt	Wet Density (gm/cc)	Input Impedance Technique (db/meter)	Power Ratio (Vector Voltmeter) (db/meter)
600	12	1.33	19.1	17.3
1000	12.8	1.38	23.7	24.9

Note: Laguana Joyuda, P.R. (Tunnel site) sample from 5'6" level.

Before turning to the conclusions, the reader is directed to Figs. 12 and 13 which show the attenuation versus frequency of a Vietnam soil (silt clay). Note a decrease in attenuation across several hundred megahertz. The rate of decrease in attenuation appears to be a function of moisture. This soil is the only one of several dozen soils observed that behaves in this fashion. No clear explanation can be given at this time for this odd behavior.

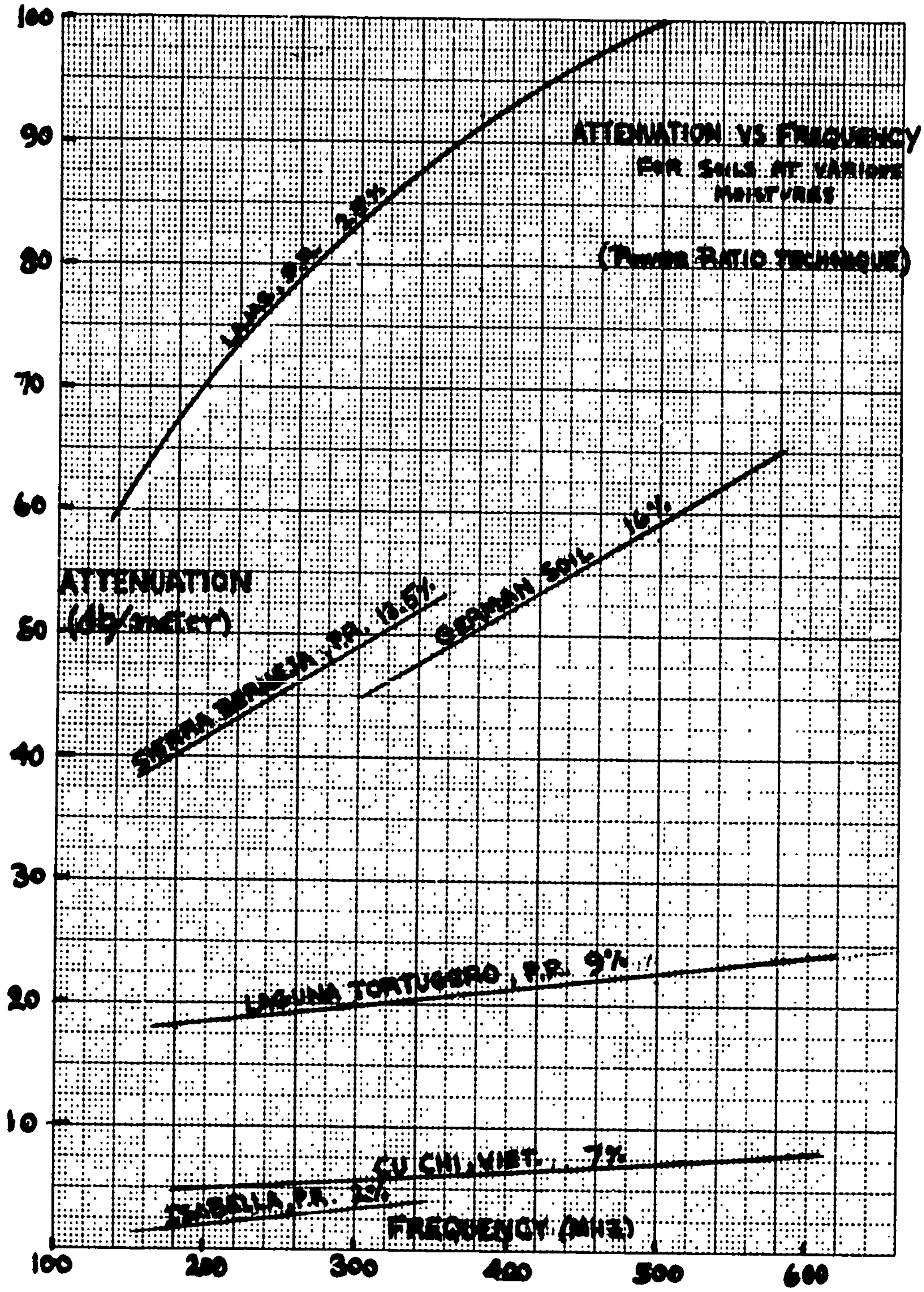


Fig. 10. Attenuation vs frequency (Power Ratio Technique).

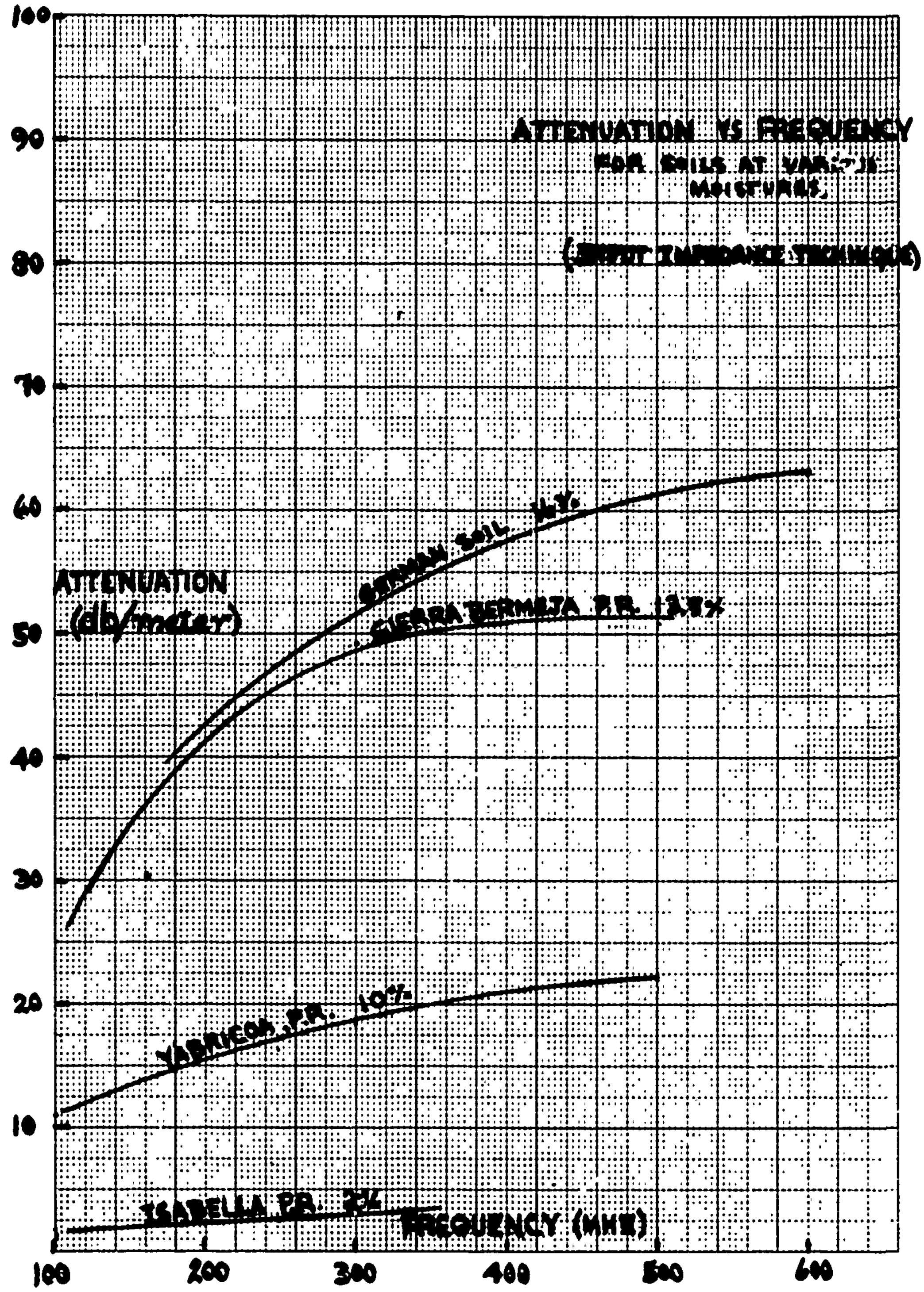


Fig. 11. Attenuation vs frequency (Input Impedance Technique).

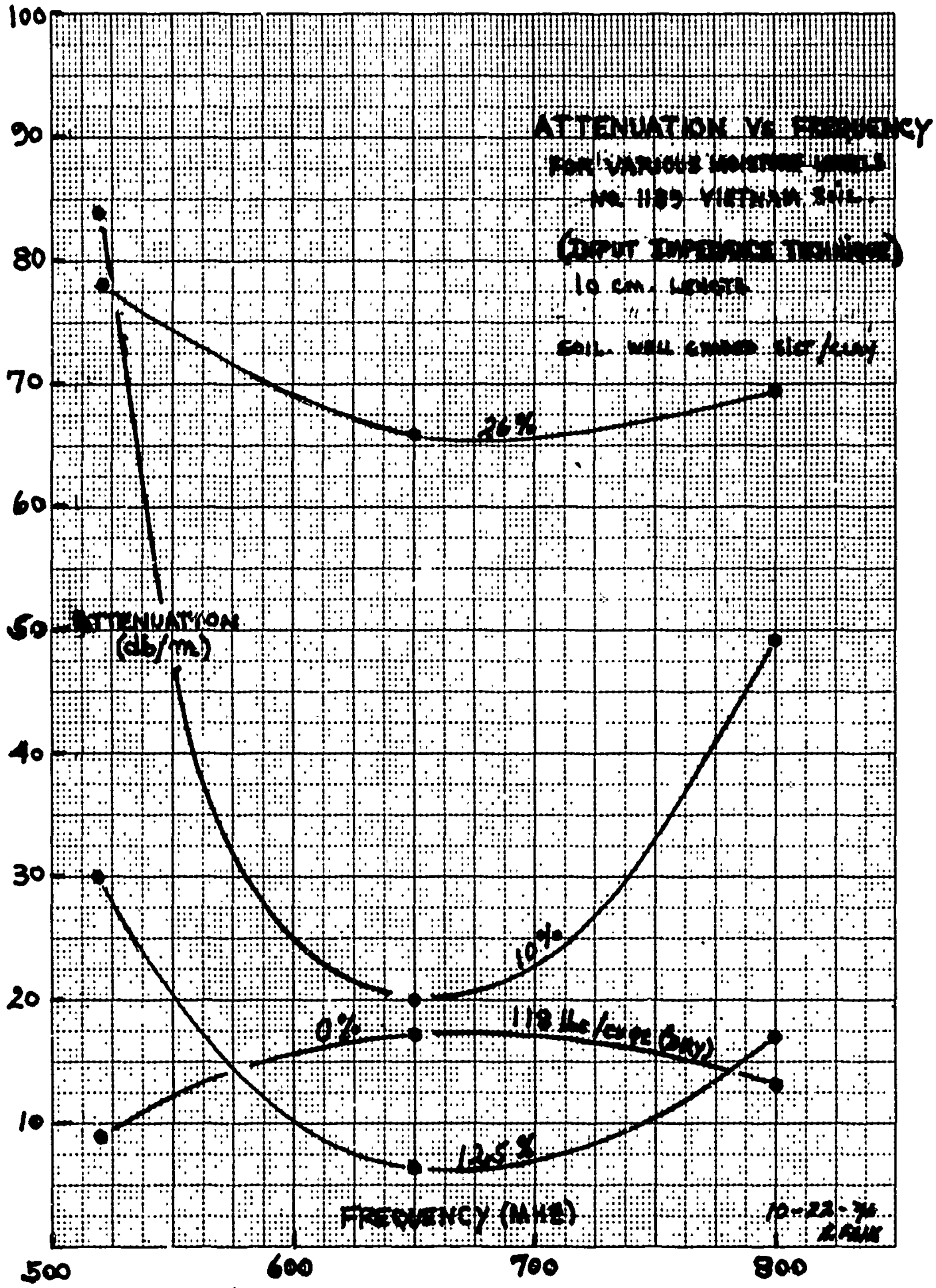


Fig. 12. Attenuation vs frequency – Vietnam Soil.

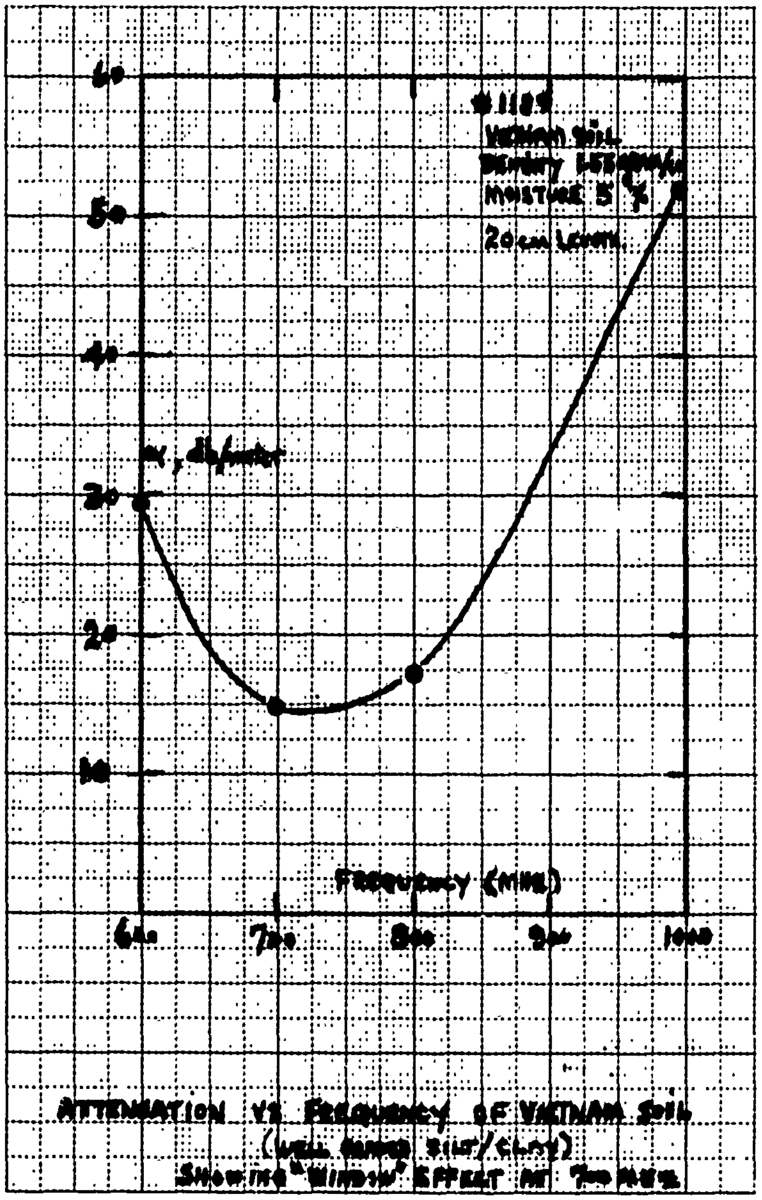


Fig. 13. "Window" effect of Vietnam soil.

III. CONCLUSIONS

7. **Conclusions.** It is concluded that:

a. An increase in moisture content of soils will generally increase attenuation. (However, there are exceptions as noted in Figs. 12 and 13 in that windows occur in the attenuation curve above 500 mHz. The transmission windows are accentuated by the percentage of moisture occurring in the soil.)

b. Correlation is obtained between the Input Impedance Technique and the Power Ratio Method (± 9%) if the density and moisture content are the same in both methods.

APPENDIX A

COMPUTER PROGRAM AND PRINTOUT

(FORTRAN IV)

CASE LAGUNA JOYUDA

VSWR(OC)= 6.50

VSWR(SC)= 2.80

L(OC)= .1600

L(SC)= .1850

LAMBDA= .1667

LENGTH= .0730

F= 1800 MCS

ZOC= 8.18664005 12.51443551

ZSC= 27.66337154 -33.19033998

ZCH= 25.37679253 1.46736654

ALPHA= 3.94698271 ALPHA F= 34.28349180

OMEGA= 1.13097337E+10

Z = 3.84337487 -.44596331 Z0 = 50.0

N	BETA	LOSSTAN	SIGMA X10^-1	EPSILON
0	6.6157	1.8526	3.6746E-04	.0198
1	28.1335	.2862	1.5626E-03	.5459
2	49.6513	.1600	2.7578E-03	1.7236
3	71.1690	.1113	3.9530E-03	3.5529
4	92.6868	.0853	5.1481E-03	6.0337
5	114.2045	.0692	6.3433E-03	9.1661
6	135.7223	.0582	7.5385E-03	12.9501
7	157.2401	.0502	8.7337E-03	17.3856
8	178.7578	.0442	9.9288E-03	22.4727
9	200.2756	.0394	1.1124E-02	28.2114
10	221.7933	.0356	1.2319E-02	34.6017
11	243.3111	.0325	1.3514E-02	41.6435
12	264.8288	.0298	1.4710E-02	49.3369
13	286.3466	.0276	1.5905E-02	57.6819
14	307.8644	.0256	1.7100E-02	66.6784

CASE LAGUNA JOYUDA

VSWR(OC)= 1.70

VSWR(SC)= 3.20

L(OC)= .2330

L(SC)= .1690

LAMBDA= .1667

LENGTH= .1730

F= 1800 MCS

ZOC= 38.38609755 20.45308475

ZSC= 15.73457789 -3.97596541

ZCH= 26.37420836 3.20765759

ALPHA= 3.43363526 ALPHA P= 29.82455586

OMEGA= 1.13097337E+10

Z = 3.43839237 -.84891835 Z0 = 50.0

N	BETA	LCSSTAN	SIGMA	EPSILON
0	-1.7691	1.4029	-8.5483E-05	-.0061
1	7.3106	1.2052	3.5325E-04	.0293
2	16.3904	.4382	7.9197E-04	.1807
3	25.4701	.2746	1.2307E-03	.4482
4	34.5499	.2007	1.6694E-03	.8316
5	43.6296	.1584	2.1082E-03	1.3311
6	52.7094	.1308	2.5469E-03	1.9466
7	61.7891	.1115	2.9856E-03	2.6780
8	70.8689	.0971	3.4243E-03	3.5256
9	79.9486	.0861	3.8631E-03	4.4891
10	89.0284	.0773	4.3018E-03	5.5686
11	98.1081	.0701	4.7405E-03	6.7642
12	107.1879	.0641	5.1793E-03	8.0758
13	116.2676	.0591	5.6180E-03	9.5033
14	125.3474	.0548	6.0567E-03	11.0469

CASE   LAGUNA JOYUDA

VSWR(OC)=   1.44

VSWR(SC)=   2.30

L(OC)=   .1700

L(SC)=   .1610

LAMBDA=   .1667

LENGTH=   .2730

F=   1800 MCS

ZOC=   35.00693333   -3.24535620

ZSC=   22.56150637   8.71889857

ZCH=   28.88312784   4.01621628

ALPHA=   3.49092984   ALPHA P=   30.32221663

OMEGA= 1.13097337E+10

Z =   2.82638338   -.80208519   ZO =   50.0

N	BETA	LOSSTAN	SIGMA	EPSILON
0	1.6159	-1.1782	7.9313E-09	-.4057
1	7.3697	1.2214	3.6294E-04	.2296
2	13.1235	.5725	6.4478E-04	.1126
3	18.8774	.3823	9.2736E-04	.2422
4	24.6312	.2893	1.2100E-03	.4283
5	30.3850	.2329	1.4927E-03	.6410
6	36.1389	.1950	1.7753E-03	.9104
7	41.8927	.1678	2.0580E-03	1.2263
8	47.6465	.1473	2.3407E-03	1.5888
9	53.4004	.1313	2.6233E-03	1.9979
10	59.1542	.1184	2.9060E-03	2.4535
11	64.9080	.1079	3.1887E-03	2.9558
12	70.6619	.0990	3.4713E-03	3.5047
13	76.4157	.0916	3.7540E-03	4.1001
14	82.1695	.0851	4.0366E-03	4.7421

CASE TEFLON

VSWR(OC)	=	43.5000	ZOC	=	1.19714719	-10.18518543		
L(OC)	=	.2450	ZCH	=	34.68348262	.81443966		
VSWR(SC)	=	39.6000	ZSC	=	8.21415972	117.07662124		
L(SC)	=	.1360	ALPHA	=	.34806703		ALPHA F =	3.02331013
DELTA L	=	.2710	OMEGA	=	3.76991124E+09			
LAMBDA	=	.5000	Z	=	2.07480608	-.09749492		
LENGTH	=	.0730	ZO	=	50.0			

FREQUENCY = 600 MCS

N	BETA	LOSSTAN	SIGMA	EPSILON
0	-3.9235	-.1788	-5.7654E-04	.0967
1	17.5942	.0396	2.5854E-03	1.9595
2	39.1120	.0178	5.7473E-03	9.6865
3	60.6297	.0115	8.9092E-03	23.2776
4	82.1475	.0085	1.2071E-02	42.7328
5	103.6653	.0067	1.5233E-02	68.0521
6	125.1830	.0056	1.8395E-02	99.2357
7	146.7008	.0047	2.1557E-02	136.2833
8	168.2185	.0041	2.4719E-02	179.1951
9	189.7363	.0037	2.7881E-02	227.9710
10	211.2541	.0033	3.1043E-02	282.6111
11	232.7718	.0030	3.4204E-02	343.1153
12	254.2896	.0027	3.7366E-02	409.4836
13	275.8073	.0025	4.0528E-02	481.7161
14	297.3251	.0023	4.3690E-02	559.8127
15	318.8428	.0022	4.6852E-02	643.7735
16	340.3606	.0020	5.0014E-02	733.5984
17	361.8784	.0019	5.3176E-02	829.2874
18	383.3961	.0018	5.6338E-02	930.8406
19	404.9139	.0017	5.9500E-02	1038.2579

CASE TEFLON

VSWR(OC) =	43.5000	ZOC =	1.19714716	-10.18518301
L(OC) =	.0950	ZCH =	35.94340633	.77646024
VSWR(SC) =	39.6000	ZSC =	9.27944658	125.69372525
L(SC) =	.2340	ALPHA =	.34430545	ALPHA F = 2.99063710
DELTA L =	.1710	OMEGA =	3.76991124E+09	
LAMBDA =	.5000	Z =	1.93237074	-.00374163
LENGTH =	.0730	Z0 =	50.0	

FREQUENCY = 600 MCS

N	BETA	LOSSTAN	SIGMA	EPSILON
0	-3.7925	-.1831	-5.5126E-04	.0903
1	17.7253	.0389	2.5765E-03	1.9888
2	39.2430	.0175	5.7042E-03	9.7515
3	60.7608	.0113	8.8319E-03	23.3763
4	82.2785	.0084	1.1960E-02	42.8692
5	103.7963	.0066	1.5087E-02	68.2243
6	125.3141	.0055	1.8215E-02	99.4435
7	146.8318	.0047	2.1343E-02	136.5269
8	168.3496	.0041	2.4471E-02	179.4744
9	189.8673	.0036	2.7598E-02	228.2860
10	211.3851	.0033	3.0726E-02	282.9618
11	232.9028	.0030	3.3854E-02	343.5017
12	254.4206	.0027	3.6982E-02	409.9056
13	275.9384	.0025	4.0109E-02	482.1743
14	297.4561	.0023	4.3237E-02	560.3063
15	318.9739	.0022	4.6365E-02	644.3023
16	340.4916	.0020	4.9492E-02	734.1634
17	362.0094	.0019	5.2620E-02	829.8881
18	383.5272	.0018	5.5748E-02	931.4770
19	405.0449	.0017	5.8875E-02	1038.9330

CASE TEFLON

VSWR(OC) = 40.0000 ZOC = 116.91876835 466.70513901

L(OC) = .2120 ZCH = 32.22449591 4.91599104

VSWR(SC) = 43.5000 ZSC = 1.19105956 -1.88485340

L(SC) = .3320 ALPHA = .15081760 ALPHA P = 1.31000165

DELTA L = .1710 OMEGA = 3.76991124E+09

LAMBDA = .5000 Z = 2.24573265 -.70151948

LENGTH = .1730 Z0 = 50.0

FREQUENCY = 600 MCS

N	BETA	LOSSTAN	SIGMA	EPSILON
0	-.3610	-1.0123	-2.2983E-05	.0007
1	8.7188	.0346	5.5513E-04	.4812
2	17.7985	.0169	1.1332E-03	2.0059
3	26.8783	.0112	1.7114E-03	4.5748
4	35.9580	.0084	2.2895E-03	8.1877
5	45.0378	.0067	2.8676E-03	12.8449
6	54.1175	.0056	3.4457E-03	18.5461
7	63.1973	.0048	4.0238E-03	25.2915
8	72.2770	.0042	4.6019E-03	33.0810
9	81.3568	.0037	5.1801E-03	41.9147
10	90.4365	.0033	5.7582E-03	51.7925
11	99.5163	.0030	6.3363E-03	62.7144
12	108.5960	.0028	6.9144E-03	74.6805
13	117.6758	.0026	7.4925E-03	87.6907
14	126.7555	.0024	8.0706E-03	101.7450
15	135.8352	.0022	8.6488E-03	116.8435
16	144.9150	.0021	9.2269E-03	132.9862
17	153.9947	.0020	9.8050E-03	150.1729
18	163.0745	.0018	1.0383E-02	168.4036
19	172.1542	.0018	1.0961E-02	[illegible]

APPENDIX B

LABORATORY PROCEDURE FOR KIRKSCETHER'S TRANSMISSION-LINE TECHNIQUE

Procedure

The sample of soil is packed into a General Radio (GR) Type 87 Air Line with the apparatus shown in Figs. 3 and 4. The center conductor extension of the apparatus is screwed to the center conductor of the air line. The line is now ready to receive the soil sample.

The soil sample is funneled into the air line a little at a time. The soil is tamped between levels of soil until the entire air line is filled (Fig. 5). The center conductor extension of the apparatus is unscrewed from the center conductor of the GR Line. This completes the filling (Fig. 6).

The air line containing the soil sample is connected to the end of the Alford slotted line with a suitable connector. The tunable probe should be inserted in the line and adjusted for maximum signal out.

The frequency to be used in the attenuation measurements is selected *on the Hewlett Packard 5105A Frequency Synthesizer.*

The Alford slotted line utilized for these experiments was 5 feet long, thus limiting the lowest frequency to 100 mHz without the use of extensions. A GR Constant-Impedance Adjustable Line (874-LK) may be used to obtain an accurate one-half-wavelength line (Fig. 1) for each frequency to be used. The one-half-wavelength lines may be obtained in the following manner:

a. The wavelength in meters can be determined by using the following formula:

$$\lambda = \frac{300}{f\ \mathrm{mHz}} = \text{wavelength in meters (assuming } C = C_o\text{)}$$

b. Before the one-half-wavelength line is attached to the slotted line, a calibrated short circuit is connected and the first null on the slotted line is located as shown in Fig. 14.

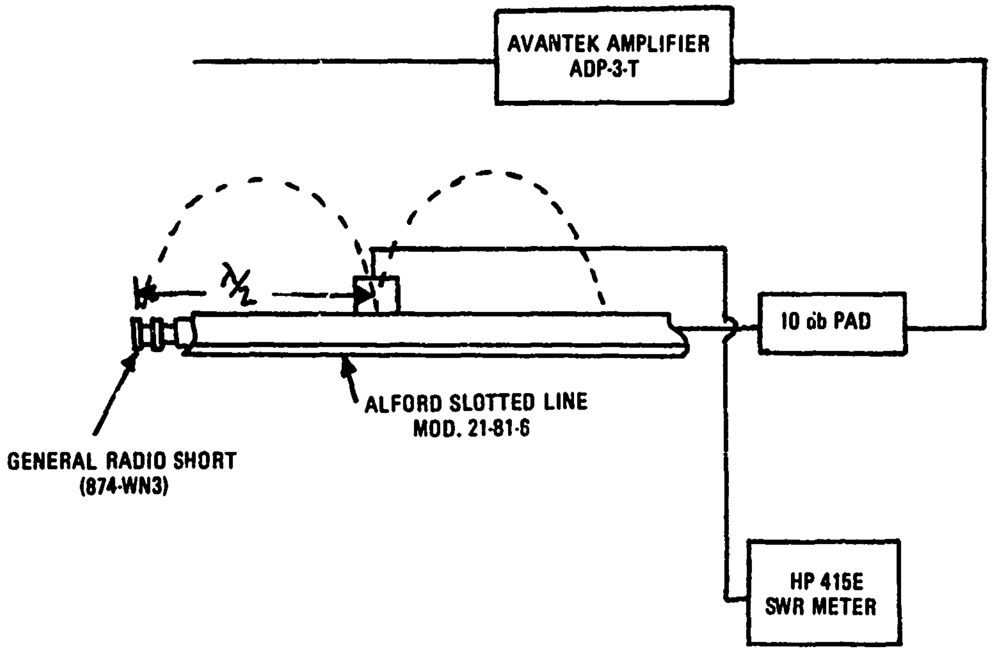


Fig. 14. Location of voltage null.

c. Once the null point has been located, the detector probe is then at one-half wavelength from the short affixed to the end of the slotted line. The short is removed, and a Telonic calibrated VSWR standard is placed on the end of the slotted line (Fig. 15). The probe is moved along the line to seek the maximum voltage; and, when this is obtained, the gain of the VSWR meter is adjusted for 0 db. The probe is then moved along to find the voltage minimum. The VSWR is indicated by the meter. This reading compared to the standard on the end of the line is the error. The error does not exceed ± 5% on a consistent basis.

d. The standard is removed and the GR short is replaced on the end of the slotted line. The probe is again moved along the line away from the short and is stopped at the first voltage minimum as shown in Fig. 16A.

e. The short is removed from the slotted line and placed on the end of the one-half-wavelength, adjustable line. The line is adjusted so that a minimum occurs at the same place as it did with the short connected directly to the slotted line as shown in Fig. 16B.

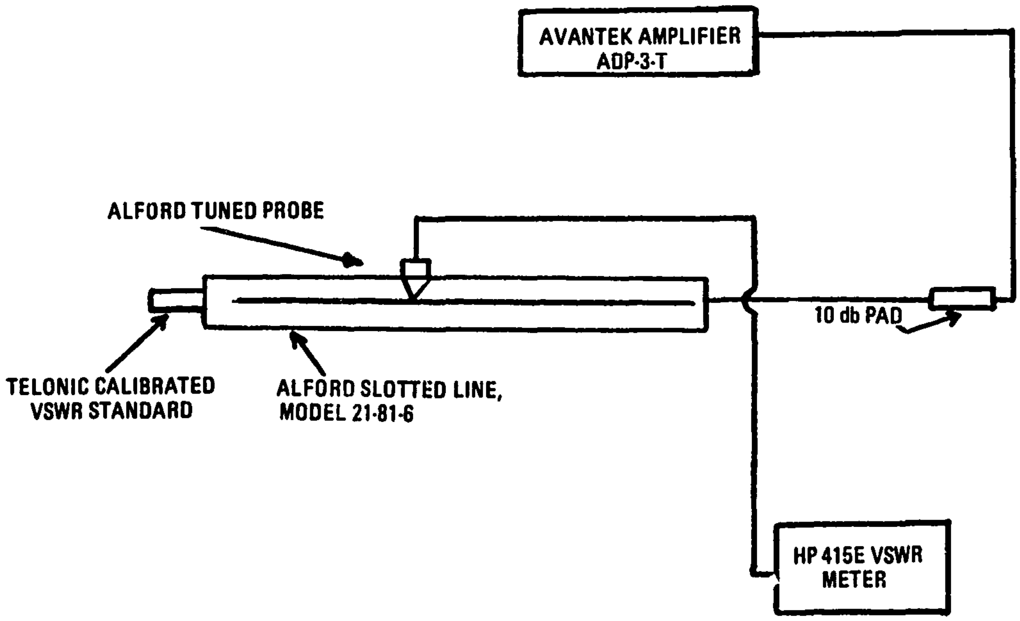


Fig. 15. System calibration.

f. The adjustable line must now be shortened by 4.6 centimeters to move the plane of the short 4.6 centimeters toward the generator as shown in Fig. 16C. The reason for this is to account for the spacing inherent in the General Radio connectors. It ensures that when the adjustable line is connected to the output end of a section of GR line filled with soil, the open or short circuit will be transformed exactly to air-soil interface (Fig. 16D).

g. It may be well at this point to recheck along the slotted line the distance between the nulls to make sure it corresponds to the right frequency. It can happen that, if the tuning knob on the probe is unintentionally turned, the distance between nulls will not be related to the generator frequency.

h. The carriage containing the probe/detector is moved along the slotted line to find the voltage maximum.

i. Once this voltage maximum is found on the VSWR meter, the gain of the instrument is increased so that the meter indicator reads a VSWR of 1.00:1 (0 db).

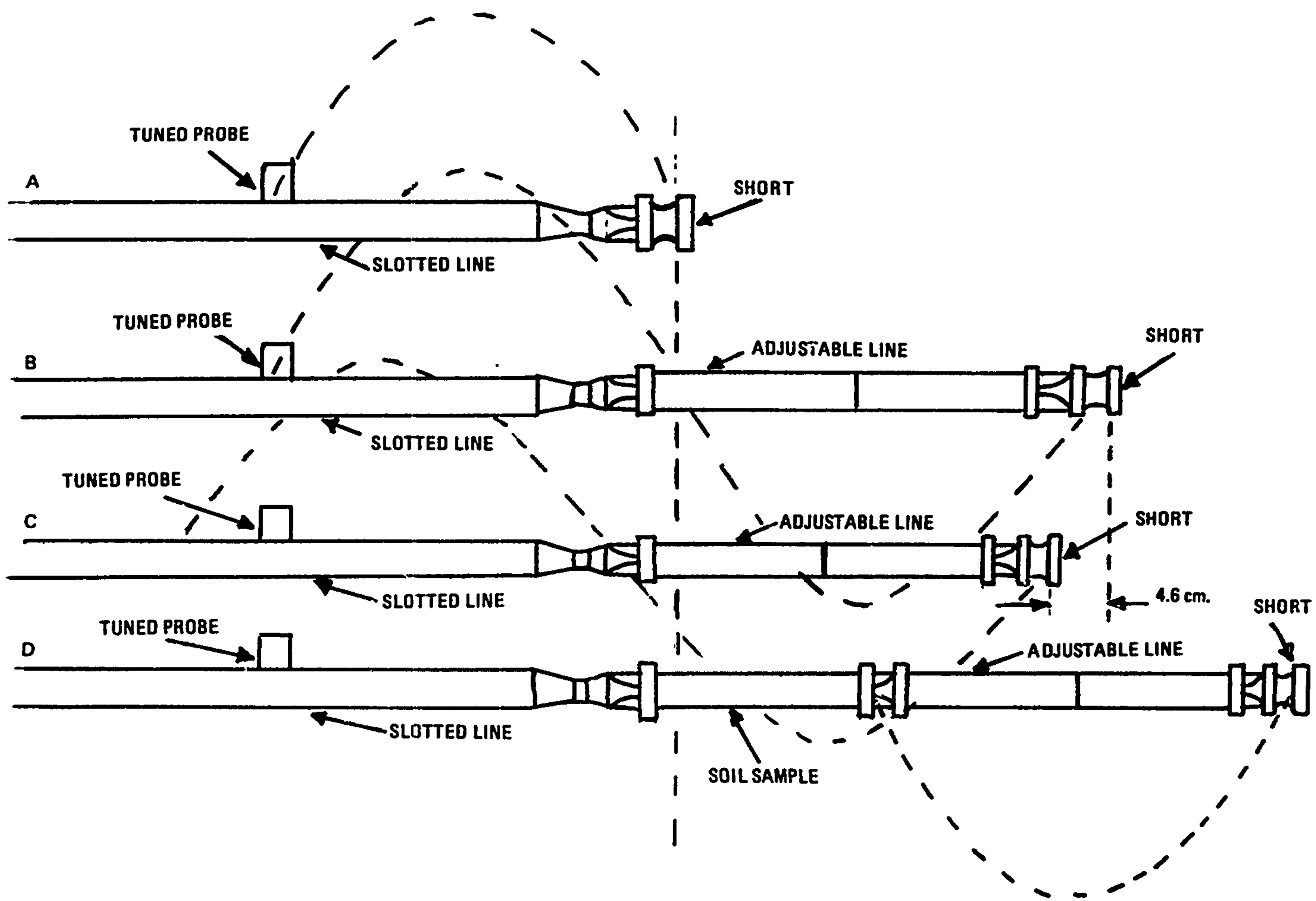


Fig. 16. Accurate adjustment procedure for short or open mode behind soil sample.

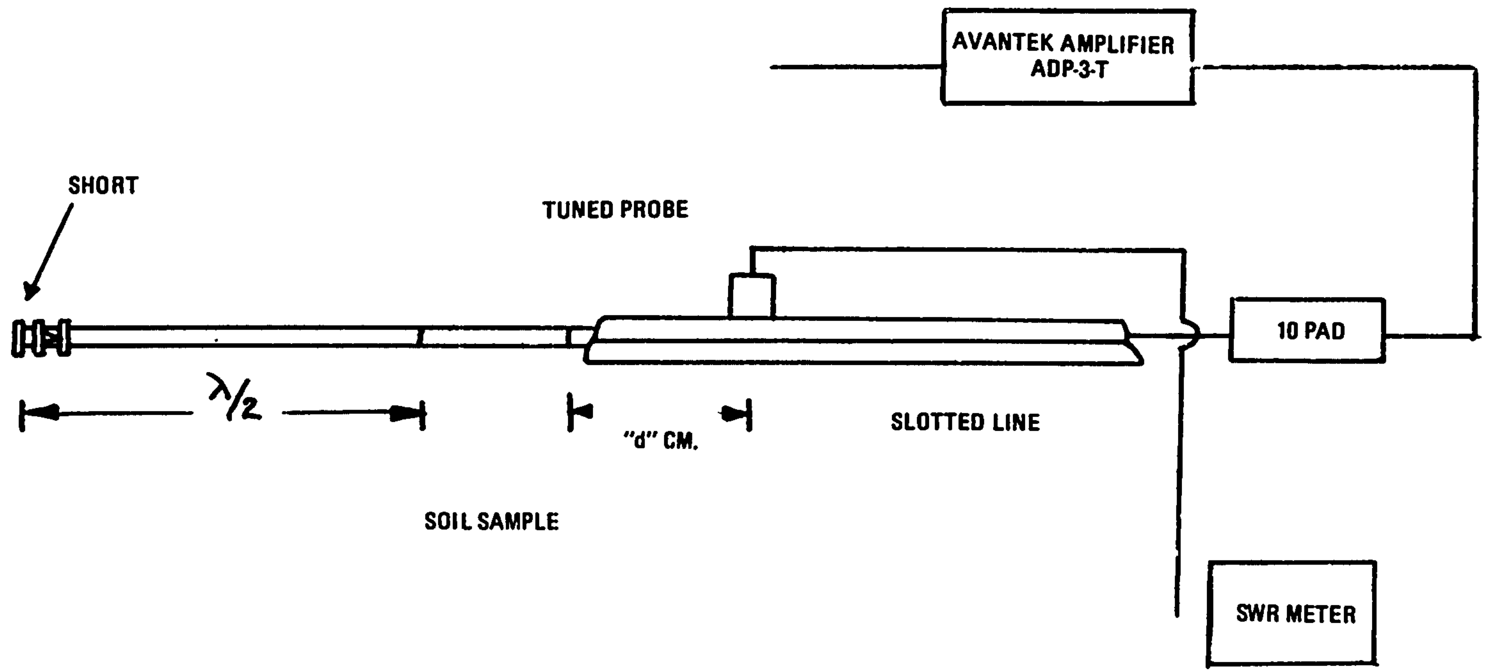


Fig. 17. Location of voltage minimum.

j. The operator should now record the following:

(1) The VSWR with the soil-filled line electrically short circuited at its output.

(2) The VSWR with the soil-filled line electrically open circuited at its output.

(3) The distance expressed in meters from a voltage minimum to the input air-soil interface with the short circuit in place (Fig. 17).

(4) The distance expressed in meters from a voltage minimum to the input air-soil interface with the open circuit in place (Fig. 17).

k. When data is obtained from a slotted-line system, one of the best aids for determining the normalized input impedance of the open- and short-circuited, soil-filled, coaxial line is the Smith Chart, and one proceeds to calculate the complex propagation constant by hand. However, this is tedious and not recommended. It is advisable to use the computer program.

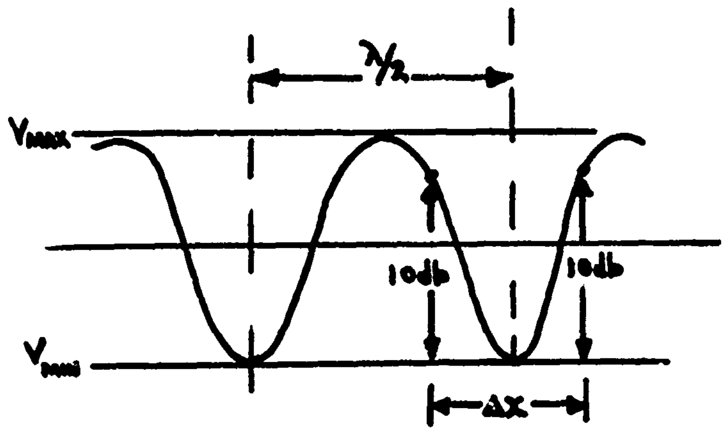
l. The Smith Chart approach works well for soils which have average to high attenuation. Normally, this produces VSWR's that are less than 10.0:1.0; however. low-loss materials such as sands produce VSWR's which are quite high.

m. Accurate, high VSWR readings for low-loss soils are best obtained by using the "Ten-Times-Minimum Method."[9] Measure the distance (d) between positions on the standing wave pattern where the voltage is 10 db above the voltage at the minimum or null point (Fig. 18). Substituting the value obtained from the slotted line at the 10-db points with the wavelength used in the following formula results in an accurate VSWR reading:

$$\text{VSWR} = \frac{3}{\pi} \left(\frac{\lambda g}{\Delta x} \right)$$

Recording the position of the 10-db points and the null point (Alford Slotted Line has a centimeter scale) in this method is important because it enables the operator to recheck his work if the need arises. This method is the same for both the open and shorted modes. The distance from the null point to the front face of the soil sample must be recorded. An example of a record is given in Table V.

[9] Hewlett Packard Operating and Service Manual for SWR meter 415E, Section 3, par. 3-29.



$$VSWR = \frac{3}{\pi}\left(\frac{\lambda_g}{\Delta x}\right)$$

Fig. 18. Ten-times minimum method for measuring high VSWR.

Table V. Data as Listed in Notebook

FREQ. (mHz)	(cm)	SHORT or OPEN	X_1 (cm)	X_2 (cm)	X_3 (cm)	Δx (cm)	VSWR	d (m)
100	300	SHORT	61.05	65.35	69.00	7.95	35.8:1.0	1.116
100	300	OPEN	23.95	25.30	26.65	2.70	105.0:1.0	1.515

where:

X_1 = 10-db point

X_2 = null point

X_3 = 10-db point

d = distance from the probe to nearest soil face

The above values of VSWR (short and open) and their respective "d" values are entered into the computer.

APPENDIX C

MODIFICATION OF TRANSMISSION-LINE EQUATION FOR INPUT IMPEDANCE

The Smith Chart greatly simplifies calculations of impedance from the measurement of VSWR and electrical length. This length can be either toward or away from the generator. Calculation of the impedance from the transmission-line equations demands that this distance from the voltage minimum be measured in wavelengths toward the generator, since this is the premise on which the equations were developed.

When a measurement of voltage minimum is made for a sample, it is ordinarily taken in distance from the air-soil interface—the distance away from the generator, that is, toward the load. The following manipulation serves to put the transmission-line equation in the proper form for transformation of impedance away from the generator.

From the literature:

$$\left.\begin{matrix} Z_{oc} \\ \\ Z_{sc} \end{matrix}\right\} = Z_o \left[\frac{Z_d \cos\beta d + j\, Z_o \sin\beta d}{Z_o \cos\beta d + j\, Z_d \sin\beta d}\right] \tag{1}$$

at a voltage minimum

$$Z_{min} = \frac{Z_o}{VSWR} \tag{2}$$

$$Z_d = Z_{min} \tag{3}$$

$$\frac{Z_{oc}}{Z_{sc}} = Z_o \left[\frac{\dfrac{Z_o}{VSWR} \cos\beta d + j\, Z_o \sin\beta d}{Z_o \cos\beta d + j\, \dfrac{Z_o}{VSWR} \sin\beta d}\right] \tag{4}$$

$$\frac{Z_{oc}}{Z_{sc}} = Z_o \left[\frac{\dfrac{1}{VSWR} \cos\beta d + j \sin\beta d}{\cos\beta d + j\, \dfrac{1}{VSWR} \sin\beta d}\right] \tag{5}$$

where, now, Z_{oc} or Z_{sc} is calculated from a measurement of the VSWR (under open or short circuit condition) and the corresponding electrical length from the voltage minimum to the input air-soil interface

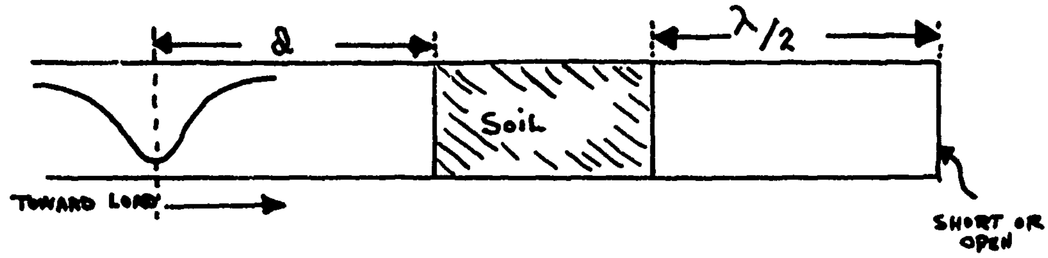
where:

$$Z_o = 50 \text{ ohms} = \text{characteristic line impedance} \tag{6}$$

and

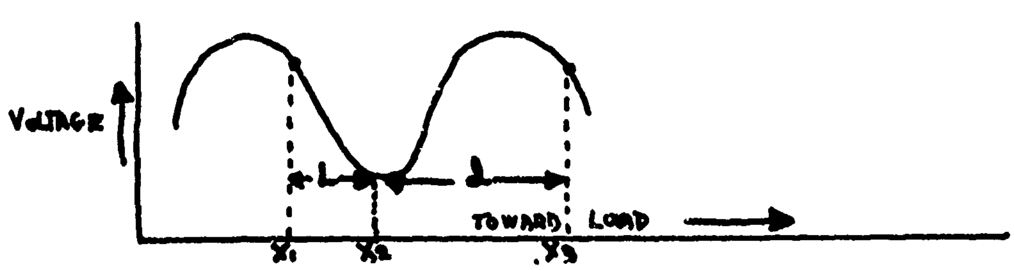
$$\beta = \frac{2\pi}{\lambda}\ ; \quad \lambda \text{ measured in meters.} \tag{7}$$

Using a diagram to illustrate:



The calibrated open circuit is placed at the end of the line and the respective open circuit parameters are measured. Next, the calibrated short circuit replaces the open circuit and the short circuit parameters are measured. The electrical length in both cases is measured away from the generator (toward the load).

Consider the sine wave below which would be a plot of the variation of voltage amplitude vs distance toward the load:



A voltage minimum is found to exist at point x_2. If the load exists at point x_3, then a corresponding point is found at x_1; and the distance from x_1 to x_3 is one wavelength. The distance measured toward the load is d; thus, the corresponding distance toward the generator, L, is

$$L = \lambda/2 - d \text{ in meters.} \tag{8}$$

Now, d in equation (5) must be replaced by L from equation (8). Thus, equation (5) becomes, with this substitution,

$$\left.\begin{matrix} Z_{oc} \\ \\ Z_{sc} \end{matrix}\right\} = Z_o \left[\frac{\frac{1}{VSWR} \cos \beta (\lambda/2-d) + j \sin \beta (\lambda/2-d)}{\cos \beta (\lambda/2-d) + j \frac{1}{VSWR} \sin \beta (\lambda/2-d)} \right] \tag{9}$$

where β = phase constant of the soil-filled line (radians/meter), and substituting for β from equation (7):

$$\left.\begin{matrix} Z_{oc} \\ \\ Z_{sc} \end{matrix}\right\} = Z_o \left[\frac{\frac{1}{VSWR} \cos \frac{2\pi}{2} (\lambda/2-d) + j \sin \frac{2\pi}{2} (\lambda/2-d)}{\cos \frac{2\pi}{\lambda} (\lambda/2-d) + j \frac{1}{VSWR} \sin \frac{2\pi}{\lambda} (\lambda/2-d)} \right] \tag{10}$$

where:

Z_{sc} = input impedance of a soil-loaded line terminated in a short circuit (ohms)

Z_{oc} = input impedance of a soil-loaded line terminated in an open circuit (ohms)

d = distance from a voltage minimum to the input terminals of the soil-loaded line (meters)

VSWR = voltage standing-wave ratio = V_{max}/V_{min}

λ = free space wavelength (meters)

Z_o = characteristic impedance of the air-filled line = 50 ohms.

APPENDIX D

SOLUTION OF LOSS TANGENT EQUATION:

$$\gamma \ell_o = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\tanh \gamma \ell_o = \sqrt{\frac{Z_{sc}}{Z_{oc}}} \qquad (1)$$

where γ = propagation constant of the soil-filled line (complex quantity).

$$\text{Let } Z_{sc} = c + jd \qquad (2)$$

$$\text{and } Z_{oc} = g + jh \qquad (3)$$

$$\text{then } \frac{Z_{sc}}{Z_{oc}} = \frac{c + jd}{g + jh} \qquad (4)$$

Rationalizing yields

$$\frac{c + jd}{g + jh} \cdot \frac{g - jh}{g - jh} = \frac{cg + hd}{g^2 + h^2} + j\,\frac{(dg - ch)}{g^2 + h^2} \qquad (5)$$

$$\text{let } a = \frac{cg + hd}{g^2 + h^2} \qquad (6)$$

$$\text{and } b = \frac{dg - ch}{g^2 + h^2} \qquad (7)$$

$$\text{then } \frac{Z_{sc}}{Z_{oc}} = a + jb \qquad (8)$$

From Fig. D-1, it can be seen that a vector, $\vec{v}$, in the complex plane can be represented by the notation $\vec{a} + \overrightarrow{jb}$

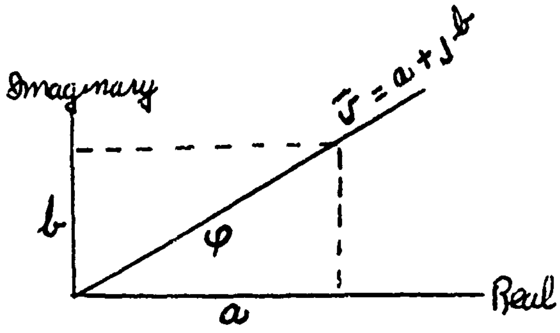


Fig. D-1

Thus a + jb can be equated to a product of an amplitude times the phase angle. If we let this phase angle be represented by $e^{i\varphi}$, then:

$$\vec{v} = \vec{a} + \overrightarrow{jb} = |a + jb|\, e^{i\varphi} \tag{9}$$

From Fig. D-1, we have

$$|\vec{a} + \overrightarrow{jb}| = \sqrt{a^2 + b^2} \tag{10}$$

$$\tan \varphi = b/a \tag{11}$$

$$\text{and } \cos \varphi = \frac{a}{\sqrt{a^2 + b^2}} \tag{12}$$

$$\text{or } \vec{a} + \overrightarrow{jb} = \sqrt{a^2 + b^2}\quad e^{i\varphi} \tag{13}$$

$$\sqrt{\vec{a} + \overrightarrow{jb}} = (a^2 + b^2)^{1/4}\ e^{i\,\varphi/2} \tag{14}$$

From Euler's theorem:

$$e^{i\varphi/2} = \cos \varphi/2 + j \sin \varphi/2 \tag{15}$$

$$\text{so that } \sqrt{\vec{a} + \overrightarrow{jb}} = (a^2 + b^2)^{1/4}\ [\cos \varphi/2 + j \sin \varphi/2\,] \tag{16}$$

$$\cos \varphi/2 = \sqrt{1/2\,(1 + \cos \varphi)} \tag{17}$$

and

$$\sin \varphi/2 = \sqrt{1/2\,(1 - \cos \varphi)} \tag{18}$$

thus:

$$\sqrt{\vec{a} + \vec{jb}} = (a^2 + b^2)^{1/4} \left[\sqrt{1/2\,(1 + \cos \varphi)} + j\sqrt{1/2\,(1 - \cos \varphi)}\right] \tag{19}$$

From equation (12):

$$\sqrt{\vec{a} + \vec{jb}} = (a^2 + b^2)^{1/4} \left\{\left[1/2\left(1 + \frac{a}{\sqrt{a^2 + b^2}}\right)\right]^{1/2} + j\left[1/2\left(1 - \frac{a}{\sqrt{a^2 + b^2}}\right)\right]^{1/2}\right\} \tag{20}$$

$$\sqrt{\vec{a} + \vec{jb}} = \left[\frac{\sqrt{a^2 + b^2}}{2} + \frac{a}{2}\right]^{1/2} + j\left[\frac{\sqrt{a^2 + b^2}}{2} - \frac{a}{2}\right]^{1/2} \tag{21}$$

Let

$$A = \left[\frac{\sqrt{a^2 + b^2}}{2} + \frac{a}{2}\right]^{1/2} \tag{22}$$

and

$$B = \left[\frac{\sqrt{a^2 + b^2}}{2} - \frac{a}{2}\right]^{1/2} \tag{23}$$

$$\sqrt{\vec{a} + \vec{jb}} = A + jB \tag{24}$$

From equations (1) and (8):

$$\tanh \gamma \ell_o = A + jB \tag{25}$$

$$\gamma \ell_o = \tanh^{-1} (A + jB) \tag{26}$$

$$\gamma \ell_o = 1/2\, \ell n \left[\frac{1 + (A + jB)}{1 - (A + jB)}\right] \tag{27}$$

Rationalizing yields

$$2\gamma\ell_o = \ell n \left[\frac{(1+A)(1-A) - B^2}{(1-A)^2 + B^2} + j \frac{\{(1+A)B + (1-A)B\}}{(1-A)^2 + B^2} \right] \tag{28}$$

$$2\gamma\ell_o = \ell n \left[\frac{1-(A^2+B^2)}{(1-A)^2+B^2} + j \frac{2B}{(1-A)^2+B^2} \right] \tag{29}$$

$$e^{2\gamma\ell_o} = \frac{1-(A^2+B^2)}{(1-A)^2+B^2} + j \frac{2B}{(1-A)^2+B^2} \tag{30}$$

By definition, $\gamma = \alpha + jB$ (31)

$$e^{2\gamma\ell_o} = e^{2(\alpha+jB)\ell_o} = e^{2\alpha\ell_o}\, e^{j2\beta\ell_o} \tag{32}$$

Using Euler's relation again yields:

$$e^{2\gamma\ell_o} = e^{2\alpha\ell_o}(\cos 2\beta\ell_o + j\sin 2\beta\ell_o) \tag{33}$$

Equating real and imaginary components of equations (30) and (33) yields:

$$e^{2\ell_o\alpha}\cos 2\ell_o\beta = \frac{1-(A^2+B^2)}{(1-A)^2+B^2} \tag{34}$$

$$e^{2\ell_o\alpha}\sin 2\ell_o\beta = \frac{2B}{(1-A)^2+B^2} \tag{35}$$

Squaring equations (34) and (35) and adding the resultant yields:

$$e^{4\ell_o\alpha}\cos^2 2\ell_o\beta + e^{4\ell_o\alpha}\sin^2 2\ell_o\beta = \left[\frac{1-(A^2+B^2)}{(1-A)^2+B^2}\right]^2 + \frac{4B^2}{[(1-A)^2+B^2]^2} \tag{36}$$

Since $\sin^2 \theta + \cos^2 \theta = 1$

$$e^{4\ell_o\alpha} = \frac{[1 - (A^2 + B^2)]^2 + 4B^2}{[(1 - A)^2 + B^2]^2} \tag{37}$$

Finally, has the solution,

$$\alpha = \frac{1}{4\ell_o} \ell n \left\{ \frac{[1 - (A^2 + B^2)]^2 + 4 B^2}{[(1 - A)^{2} + B^2]^2} \right\} \tag{38}$$

where α = attenuation constant of soil-filled line (db/meter) and

A and B are functions of a and b which, in turn, are functions of Z_{sc} and Z_{oc}.

$$\alpha = \frac{1}{4\ell_o} \ell n \; [\, f (Z_{sc}, Z_{oc}) \,] \quad \text{which is the result shown.} \tag{39}$$

In order to solve for β, we again make use of equations (34) and (35).

If we divide equation (34) into (35), we have,

$$\frac{e^{2\ell_o\alpha} \sin 2 \ell_o \beta = \dfrac{2B}{(1 - A)^2 + B^2}}{e^{2\ell_o\alpha} \cos 2 \ell_o \beta = \dfrac{1 - (A^2 + B^2)}{(1 - A)^2 + B^2}} \tag{40}$$

$$\tan 2 \ell_o \beta = \frac{2B}{1 - (A^2 + B^2)} \tag{41}$$

$$\beta = \frac{1}{2\ell_o} \tan^{-1} \frac{2B}{1 - (A^2 + B^2)} \tag{42}$$

However, since the tangent is a multivalued function, equation (42) presents only the first-order solution and not necessarily the correct solution. Thus, the solution for β is not unique, and the correct value must be determined by other measurements. Values obtained from the equation,

$$\beta_n = \frac{1}{2\ell_o} \tan^{-1} \left[\frac{2B}{1 - (A^2 + B^2)} + n\pi \right] \tag{43}$$

$$\text{or} \quad \beta_n = \frac{1}{2\ell_o} \left\{ \tan^{-1} \left[g(Z_{sc}, Z_{oc}) \right] + n\pi \right\} \tag{44}$$

are tabulated for several values of n as an output of the computer program. Where $g(Z_{sc}, Z_{oc})$ are known functions, β_n is the n^{th} solution of a multivalued function and ℓ_o is the physical length of the soil-filled section of coaxial line.